Problem Set 2B (Last revised 9/23/2008)

1. An aerial camera photographs a car travelling along a straight road on flat ground towards a junction. Before the junction there are two warning signs at distances of 4km and 2km from the junction. On the film, the signs are 1cm and 3cm from the junction and the car is 3/7cm from the junction. How far is the car from the junction on the ground?

   Proof:

1.19 Define the cross ratio of 4 ordered points P_1, P_2, P_3, P_4 in general position in $\mathbf{P}^1$ to be the image of P_4 under the projective transformation sending P_i to E_i for $i = 1, 2, 3$. Show that P_1, P_2, P_3, P_4 are projectively equivalent (as an ordered set) to P_1', P_2', P_3', P_4' if and only if the cross ratio of the sets agrees. Generalize this to show that n+3 ordered points $P_1, P_2, P_3, \ldots, P_{n+3}$ in general position in $\mathbf{P}^n$ are projectively equivalent to $P_1', P_2', P_3', \ldots, P_{n+3}'$ if and only if the cross ratio of Q_1, Q_2, Q_3, Q_i equals that of Q_1', Q_2', Q_3', Q_i' for $i = 4, \ldots, n+3$ where the points Q_i are the preimage of the points P_i under the map from $\mathbf{P}^1$ given by the unique rational normal curve through the P_i (similarly for P_i').

Proof:

1.21 Let P be a point in the projective plane and let W_P be the hyperplane in $\mathbf{P}^5$ defined by the condition that the quadratic polynomial $\lambda_0 Z_0^2 + \lambda_1 Z_1^2 + \lambda_2 Z_2^2 + \lambda_3 Z_0 Z_1 + \lambda_4 Z_0 Z_2 + \lambda_5 Z_1 Z_2$ vanishes at P. If P_i are 5 points in $\mathbf{P}^2$ with no 4 collinear show that the hyperplanes W_{P_i} intersect in a single point. Show that this mean s that there is a unique plane conic curve through the 5 points.

Proof: