1. In this problem we let \( C \) be the straight line from 0 to \( 2 + i \).

(a) Evaluate \( \int_C (z^2 + \bar{z}) \, dz \).

(b) Find an “ML” bound for \( \left| \int_C \frac{e^{2iz}}{(z + 1)(z + 4)} \, dz \right| \).

2. In (a) and (b) below we let \( f(z) = \frac{1}{3 - z} \).

(a) Find the Taylor series of \( f(z) \) with center 1. In what region does this series represent \( f(z) \)?

(b) Find a Laurent series of \( f(z) \) with center 1 which is distinct from the Taylor series you found in (a). In what region does this Laurent series represent \( f(z) \)?

(c) Give an example of a function \( g(z) \) which is analytic in \( \mathbb{C} \) except at three singular points and which has exactly three distinct Laurent series in powers of \( z \) (and \( 1/z \)), one of which is a Taylor series.

3. Use the Residue Theorem to evaluate \( \int_0^\pi \frac{d\theta}{5 - 4 \cos \theta} \).

4. (a) We know that \( \sin n\pi = 0 \) for \( n = 0, \pm 1, \pm 2, \ldots \). Show that each of these zeros of \( \sin z \) is of order 1.

In the remainder of this problem we define \( f(z) = \frac{z \sin^2 z}{e^z \sin 4z} \).

(b) Find all singular points of \( f(z) \) and for each, if it is isolated, classify it as removable, essential, or a pole of specified order. Explain your reasoning.

(c) Find all zeros of this function, after any removable singularities have been removed, and the order of each. Explain your reasoning.

(d) If \( f(z) \) is expanded in a Taylor series about the center \( z = i \), what will the radius of convergence of the series be? **Do not attempt to calculate the series.**

5. Use the Residue Theorem to evaluate \( \int_{-\infty}^{\infty} \frac{x \sin 2x}{(x^2 + 1)(x^2 + 9)} \, dx \). Justify all steps of your calculation.

6. Suppose that the function \( f(z) \) is analytic in a domain \( D \) which contains the closed disk \( |z| \leq R \). Show formally from Cauchy’s integral formula that, in the interior of this disk, \( f(z) \) is equal to the sum of its Taylor series with center \( z = 0 \). Here “formally” means that you are not asked to give a proof that your manipulations are justified.

**FORMULAS**

\[
\sin w = \sum_{n=0}^{\infty} \frac{(-1)^n w^{2n+1}}{(2n+1)!} \quad \cos w = \sum_{n=0}^{\infty} \frac{(-1)^n w^{2n}}{(2n)!}
\]

\[
e^w = \sum_{n=0}^{\infty} \frac{w^n}{n!} \quad \frac{1}{1-w} = \sum_{n=0}^{\infty} w^n, \quad |w| < 1
\]

\[
\text{Res}_{z=a} f(z) = \frac{1}{(n-1)!} \lim_{z \to a} \frac{d^{n-1}}{dz^{n-1}}[(z-a)^n f(z)]
\]

\[
f^{(n)}(z) = \frac{n!}{2\pi i} \oint \frac{f(\zeta)}{(\zeta - z)^{n+1}} \, d\zeta
\]