The Predator-Prey System

(Lotka-Volterra equations)

If x is the prey species and y the predator, then the Lotka-Volterra equations are

$$\mathbf{x}' = x(a - \alpha y), \qquad y' = y(-c + \gamma x)$$
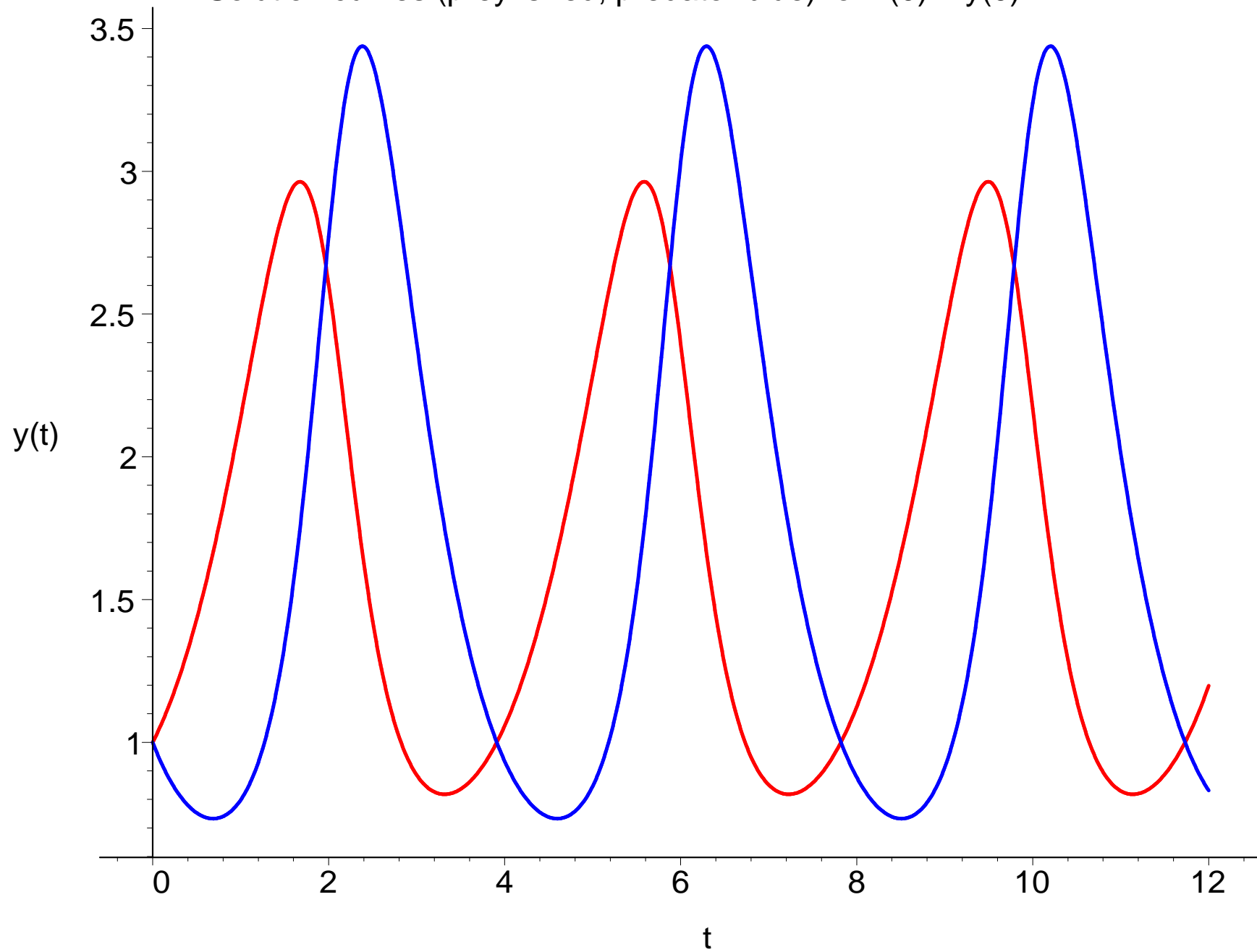
where a, c, α, and γ are positive constants. For the graphs that follow we take $a = 1.4$, $c = 2.0$, $\alpha = 0.8$, and $\gamma = 1.2$.

We plot the phase plane, followed by solution curves for two different initial conditions. It is a good exercise to trace a solution around a trajectory in the phase plane, noticing when x and y increase and decrease, and to compare the result with the soltuion curves. In each case you should see that the predator lags behind the prey by about 1/4 of a period.

Slope field, nullclines, and trajectories: predator-prey model
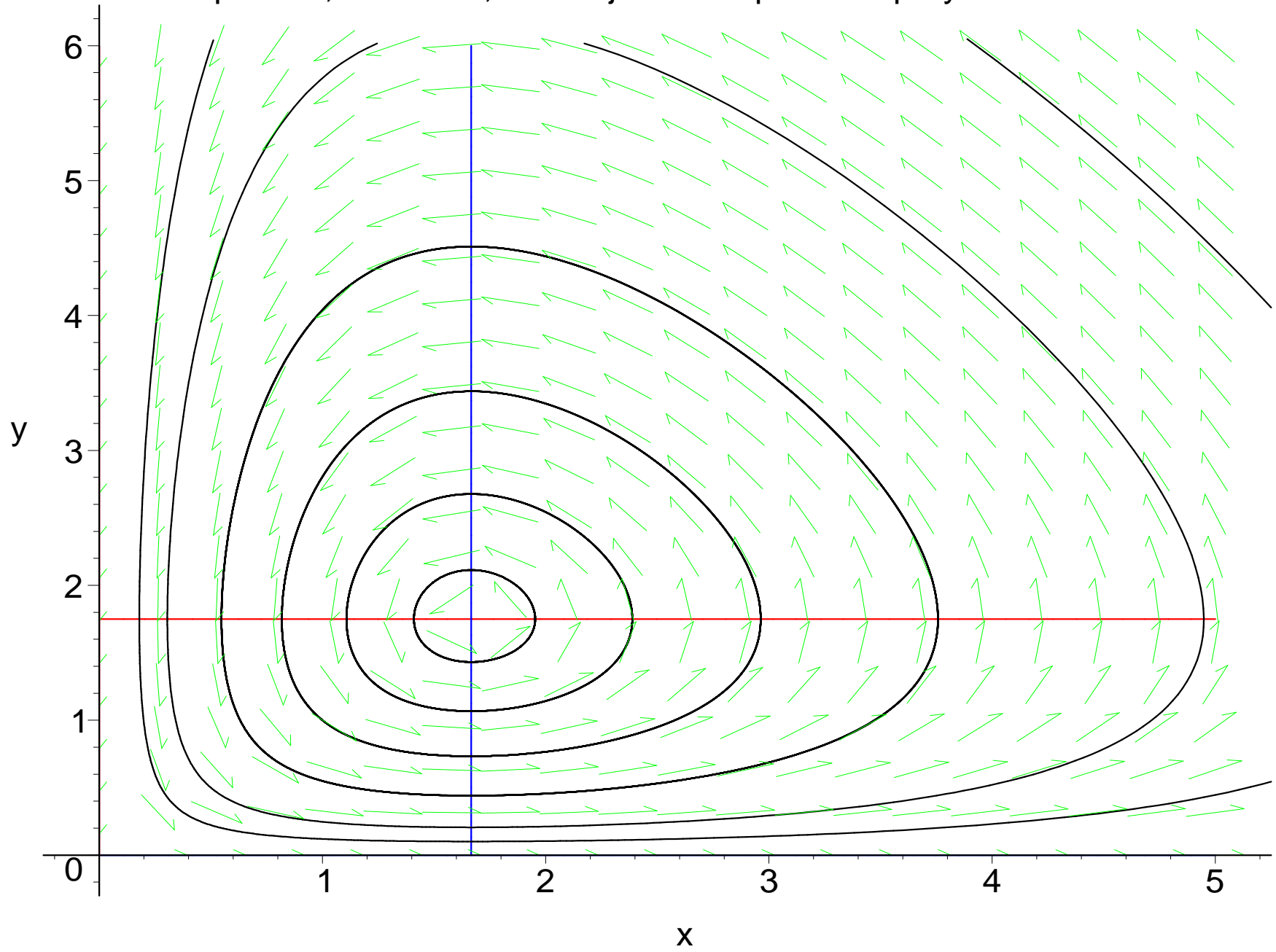
Solution curves (prey is red, predator blue) for x(0) = y(0) = 1

Solution curves (prey is red, predator blue) for x(0) = y(0) = 0.4