Competing Species Models

The general equations for competing species are of the form

$$x' = x(\epsilon_1 - \sigma_1 x - \alpha_1 y) \qquad y' = y(\epsilon_2 - \sigma_2 y - \alpha_2 x),$$

where x and y are the populations of two species in some environment, ϵ_1 and ϵ_2 are positive constants representing the growth rates of these species when there is no competition, σ_1 and σ_2 are positive constants representing intra-species competition for resurces, and α_1 and α_2 are positive constants representing inter-species competition.

Depending on the values of the parameters, these models may behave in various ways. We will concentrate on two of these: *strong competition*, in which one or the other species eventually dies out, with the winner determined by the initial conditions, and *weak competition*, in which both species can survive. We will make choose values for the parameters of these two types of models which keep our computations simple, not values corresponding to realistic models.

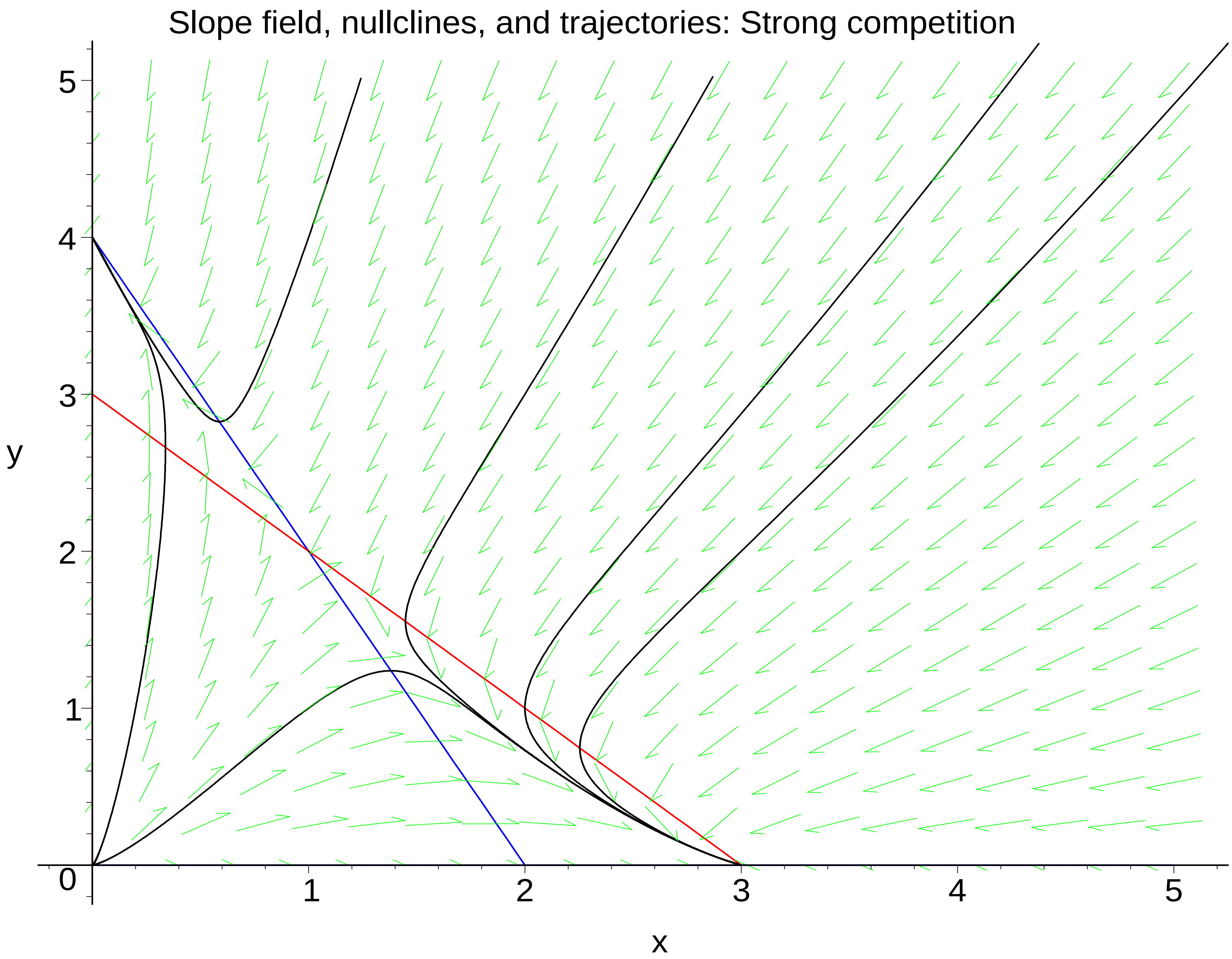
Our model for strong competition is

$$x' = x(3 - x - y), \qquad y' = y(4 - y - 2x),$$

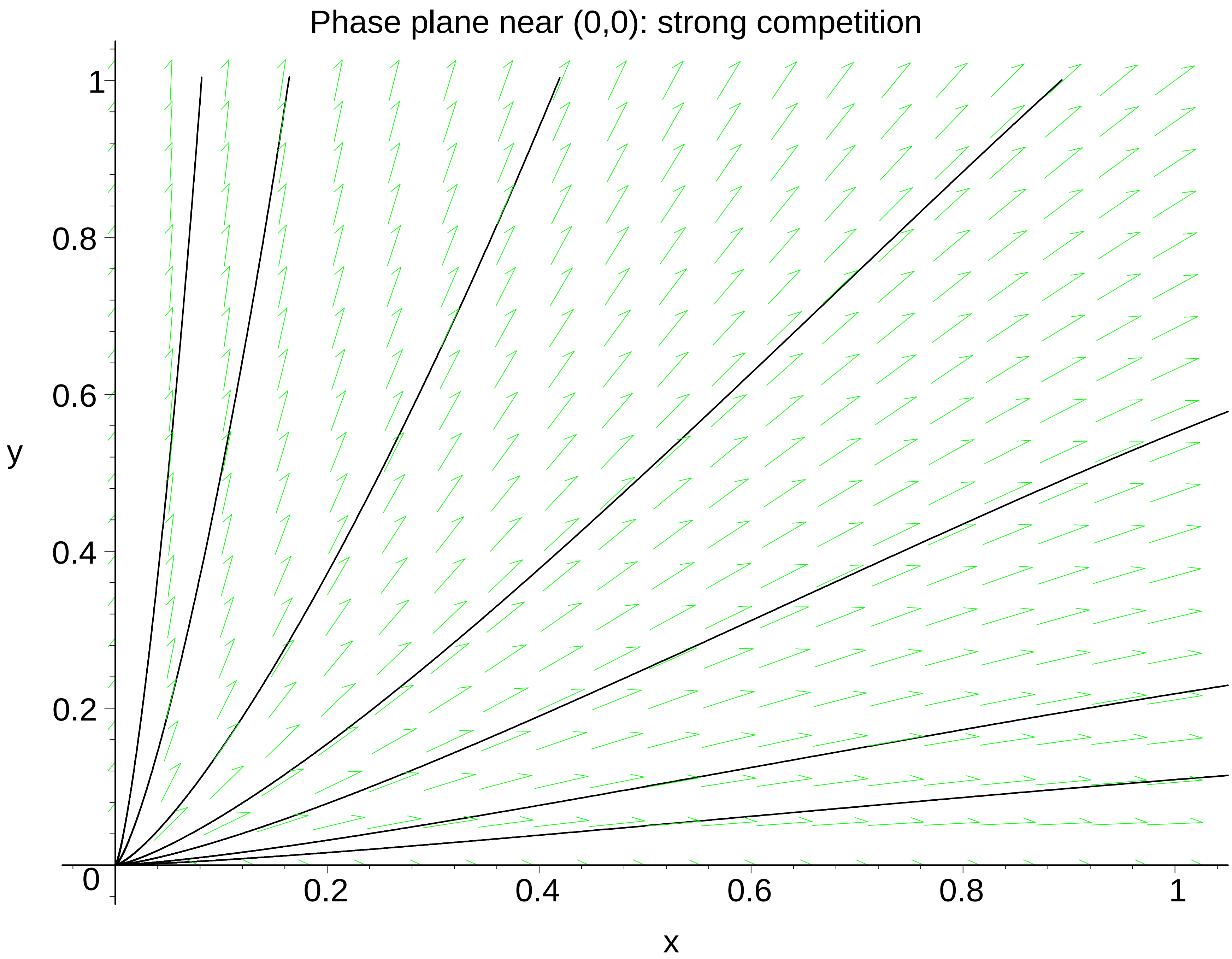
and for weak competition we take

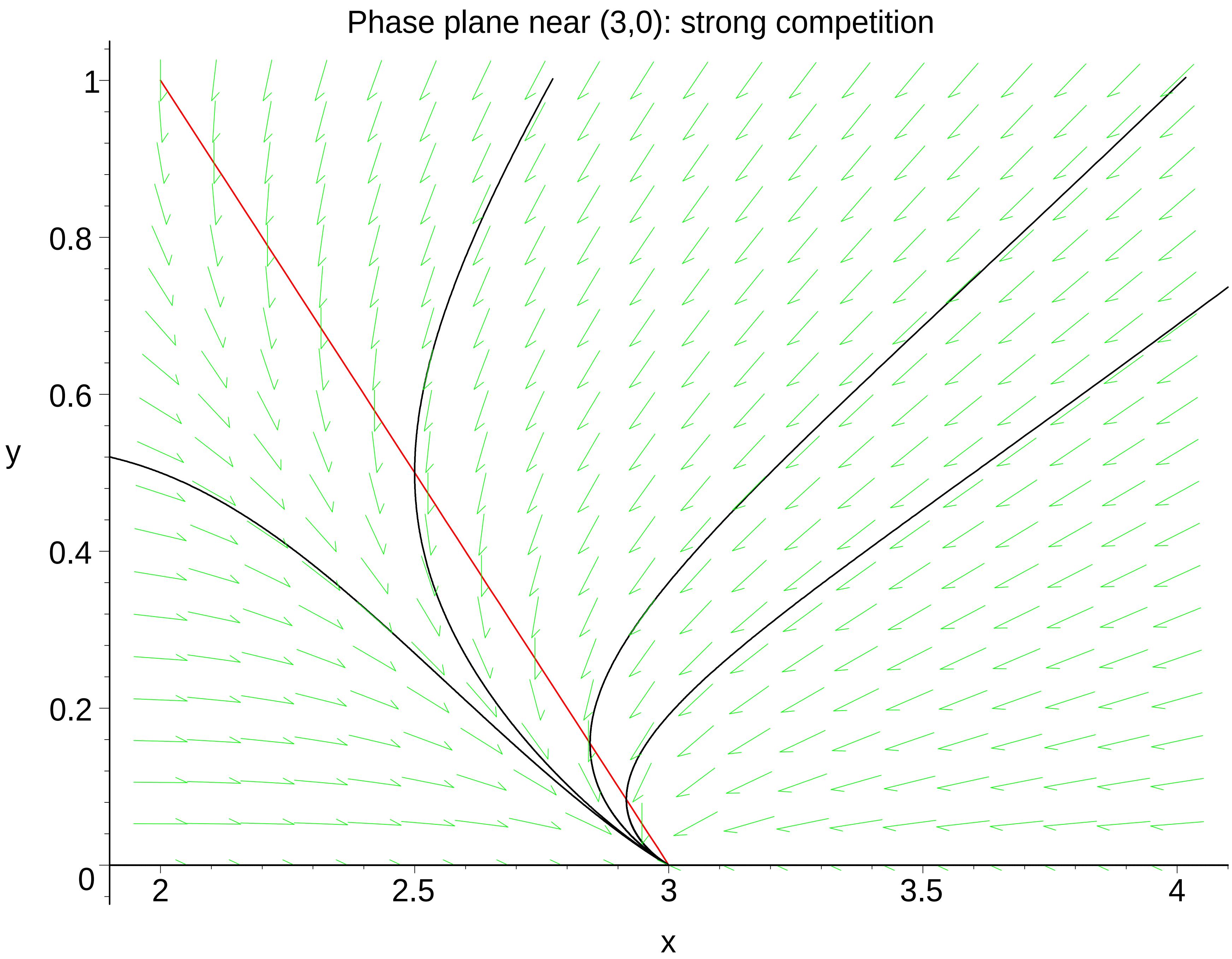
$$x' = x(4 - y - 2x), \qquad y' = y(3 - x - y).$$

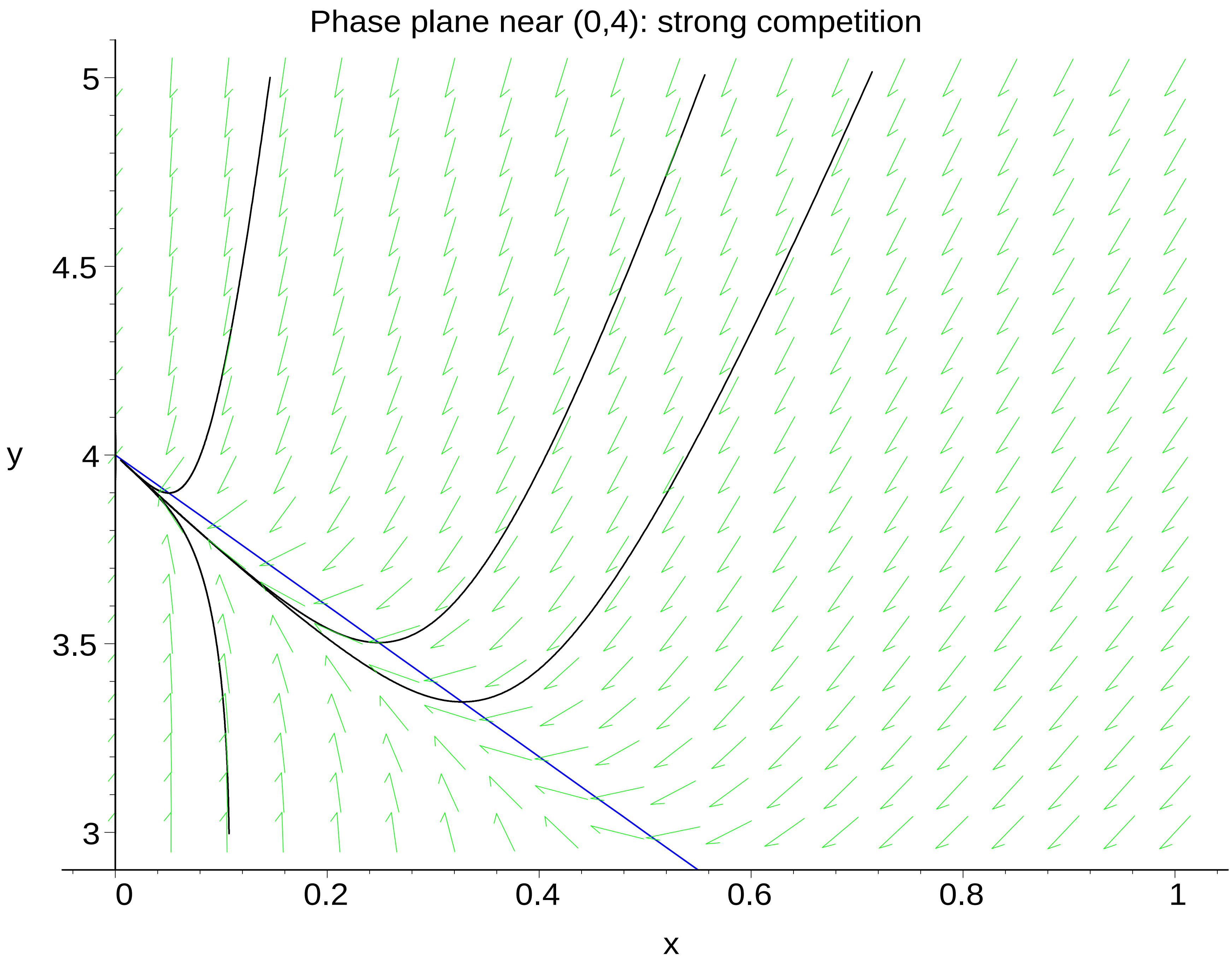
For each of these we will show a plot of the phase plane (or rather, the first quadrant of the phase plane, since only nonnegative values of populations can be relevant), as well as "close-up" view of the phase plane near some or all of the critical points. We show also the nullclines; the red nullclines are those on which the direction field is vertical, the blue those on which the direction field is horizontal. (The axes are also nullclines, but we do not color them.) See the class notes for a derivation of the types of the critical points, linearization computations, discussion of nullclines, etc.



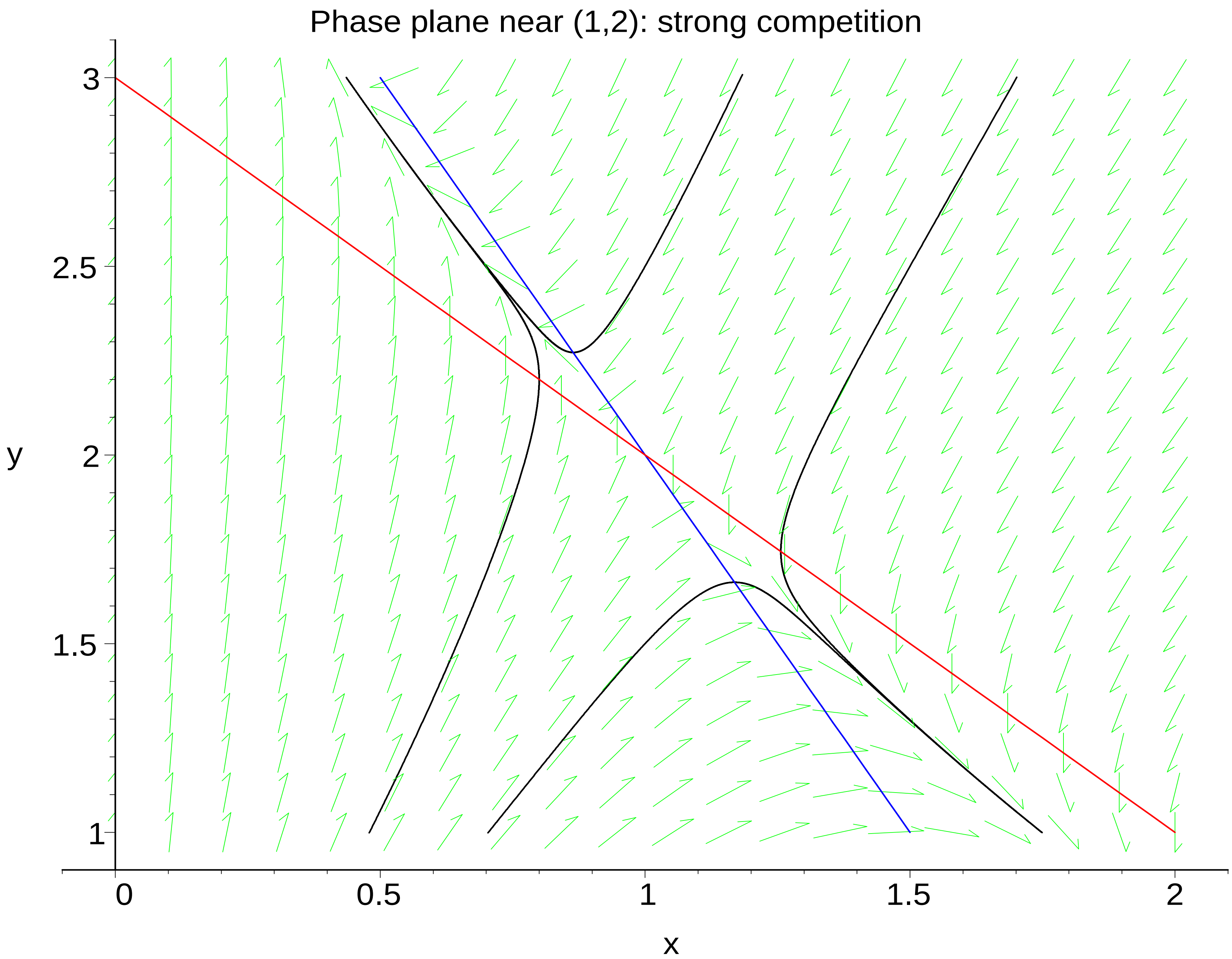
Slope field, nullclines, and trajectories: Strong competition
y
x
0
1
2
3
4
5
1
2
3
4
5


Phase plane near (0,0): strong competition
y
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
x

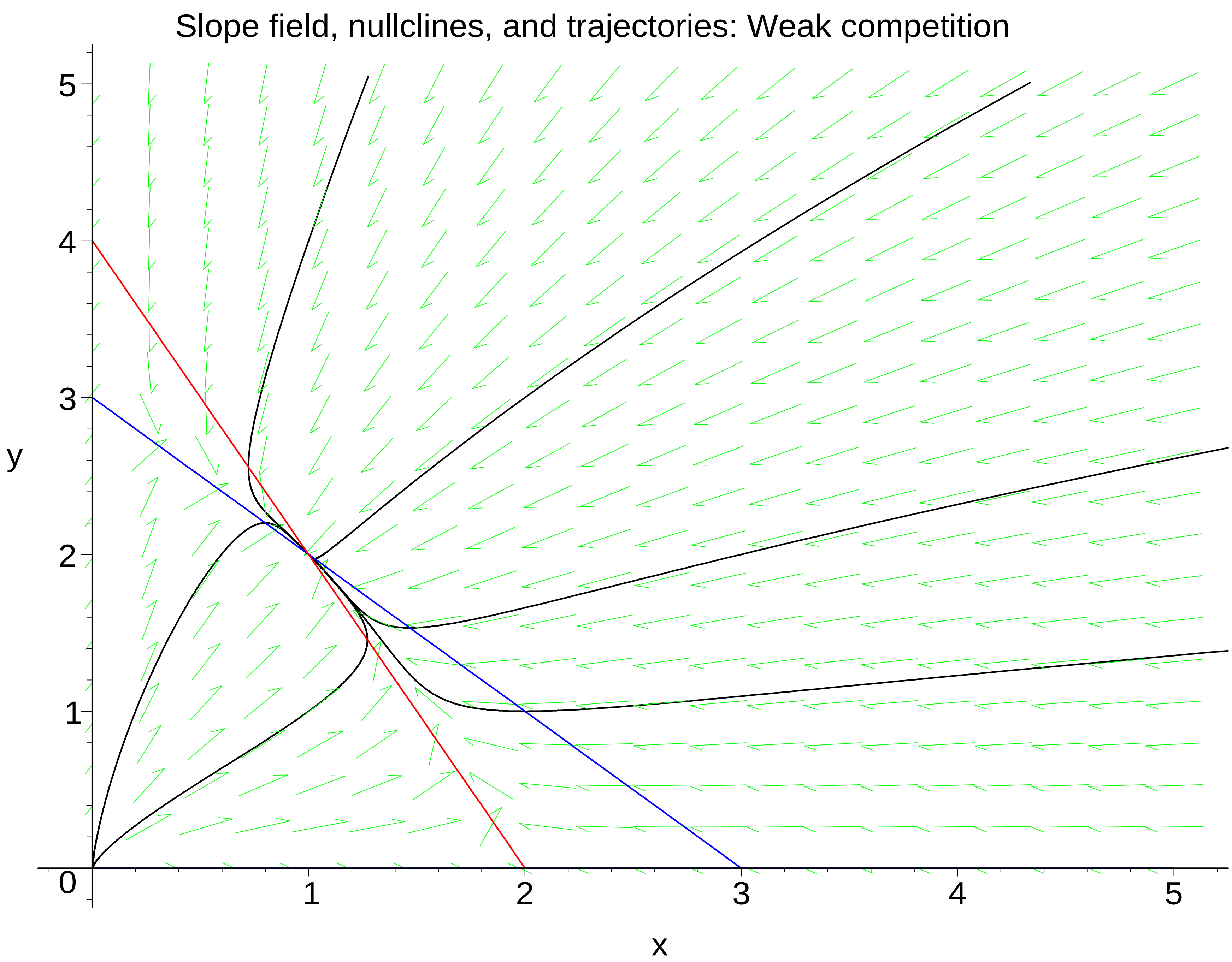

Phase plane near (3,0): strong competition
y
x
0
0.2
0.4
0.6
0.8
1
2
2.5
3
3.5
4



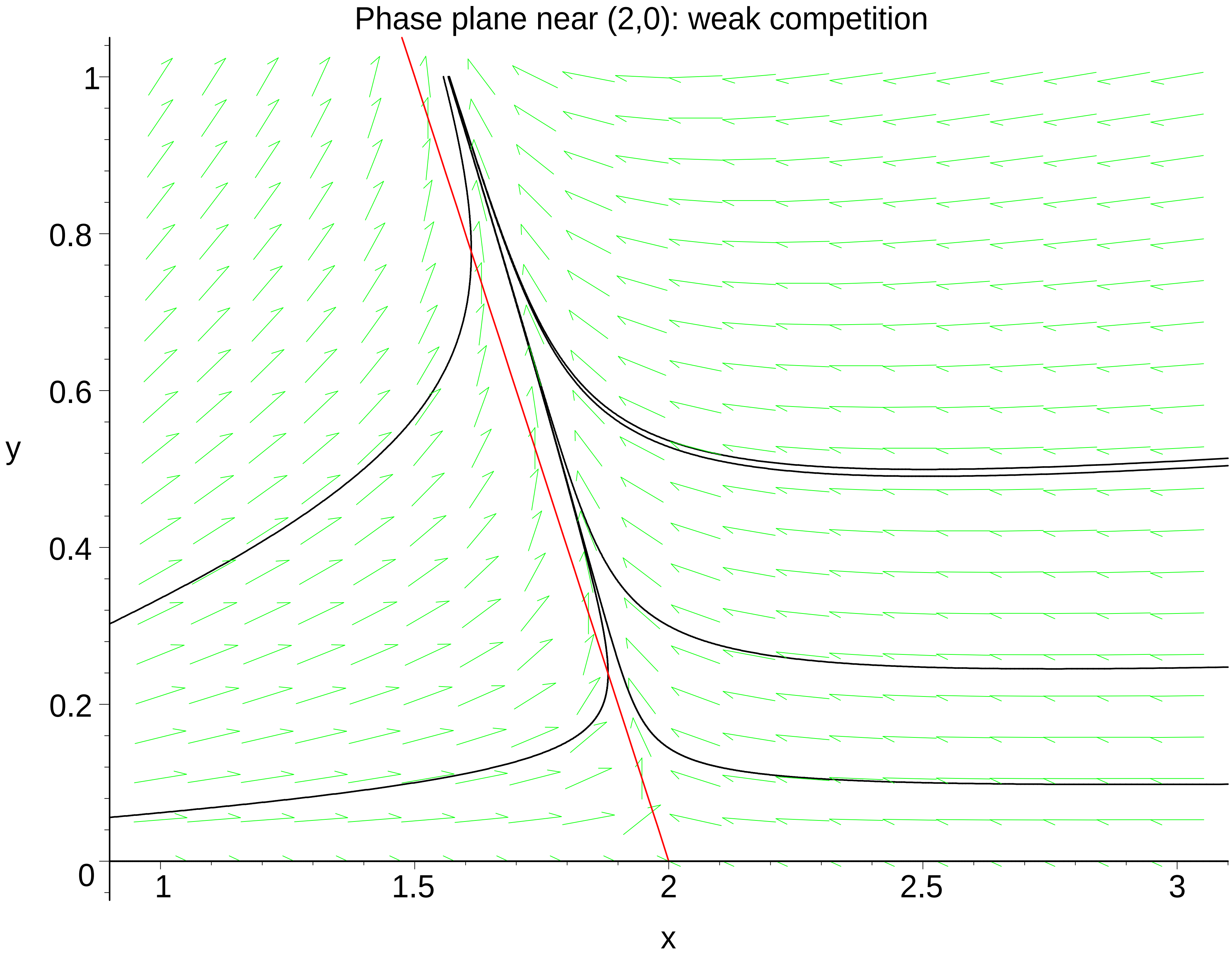
Phase plane near (0,4): strong competition
y
x
5
4.5
4
3.5
3
0
0.2
0.4
0.6
0.8
1

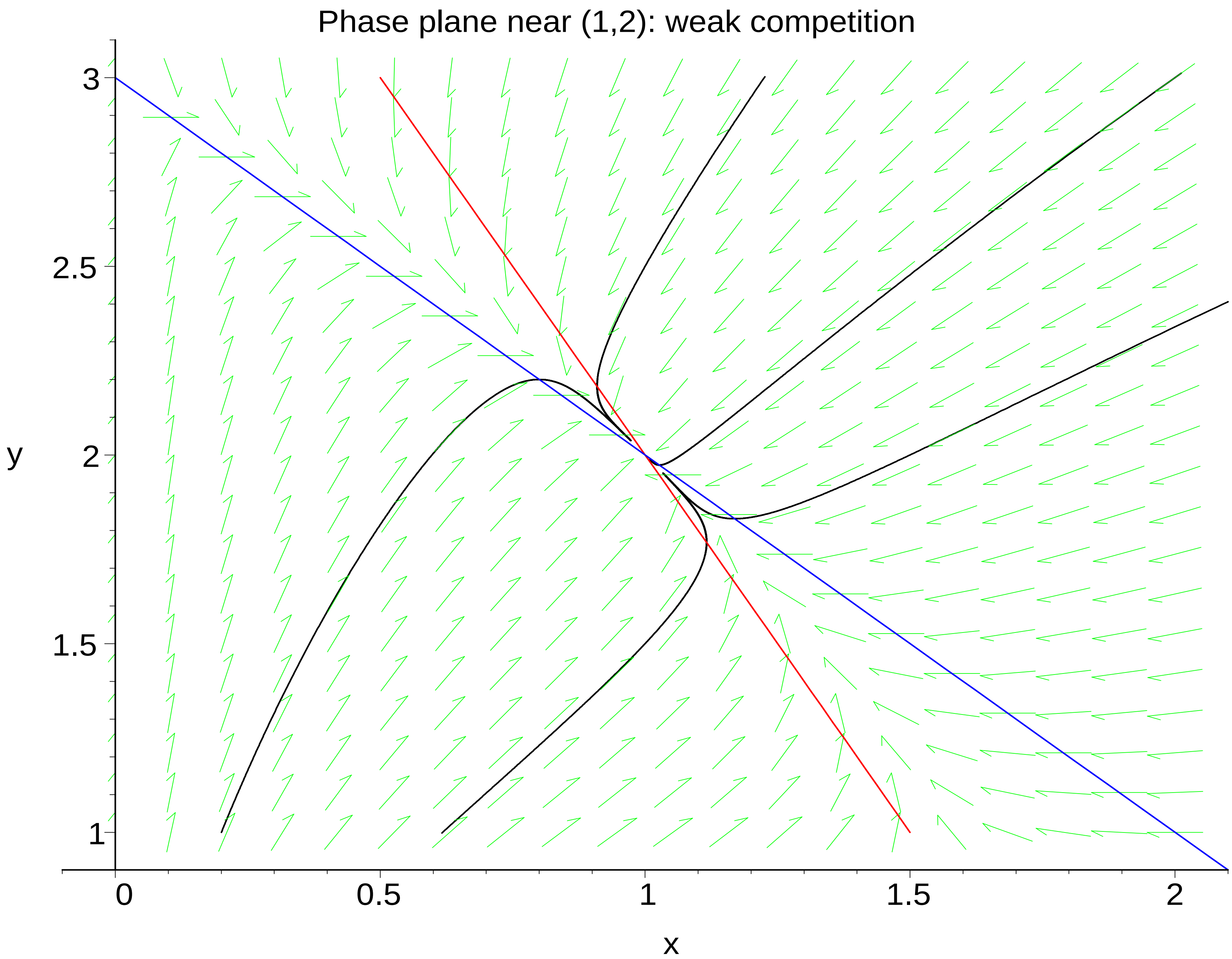


Phase plane near (1,2): strong competition
y
3
2.5
2
1.5
1
0
0.5
1
1.5
2
x



Slope field, nullclines, and trajectories: Weak competition
y
x
0
1
2
3
4
5
1
2
3
4
5


Phase plane near (2,0): weak competition
y
1
0.8
0.6
0.4
0.2
0
x
1
1.5
2
2.5
3


Phase plane near (1,2): weak competition
y
x
3
2.5
2
1.5
1
0
0.5
1
1.5
2