The Van der Pol oscillator

The Van der Pol oscillator is governed by the second order equation

$$x'' - \epsilon(1 - x^2)x' + x = 0.$$

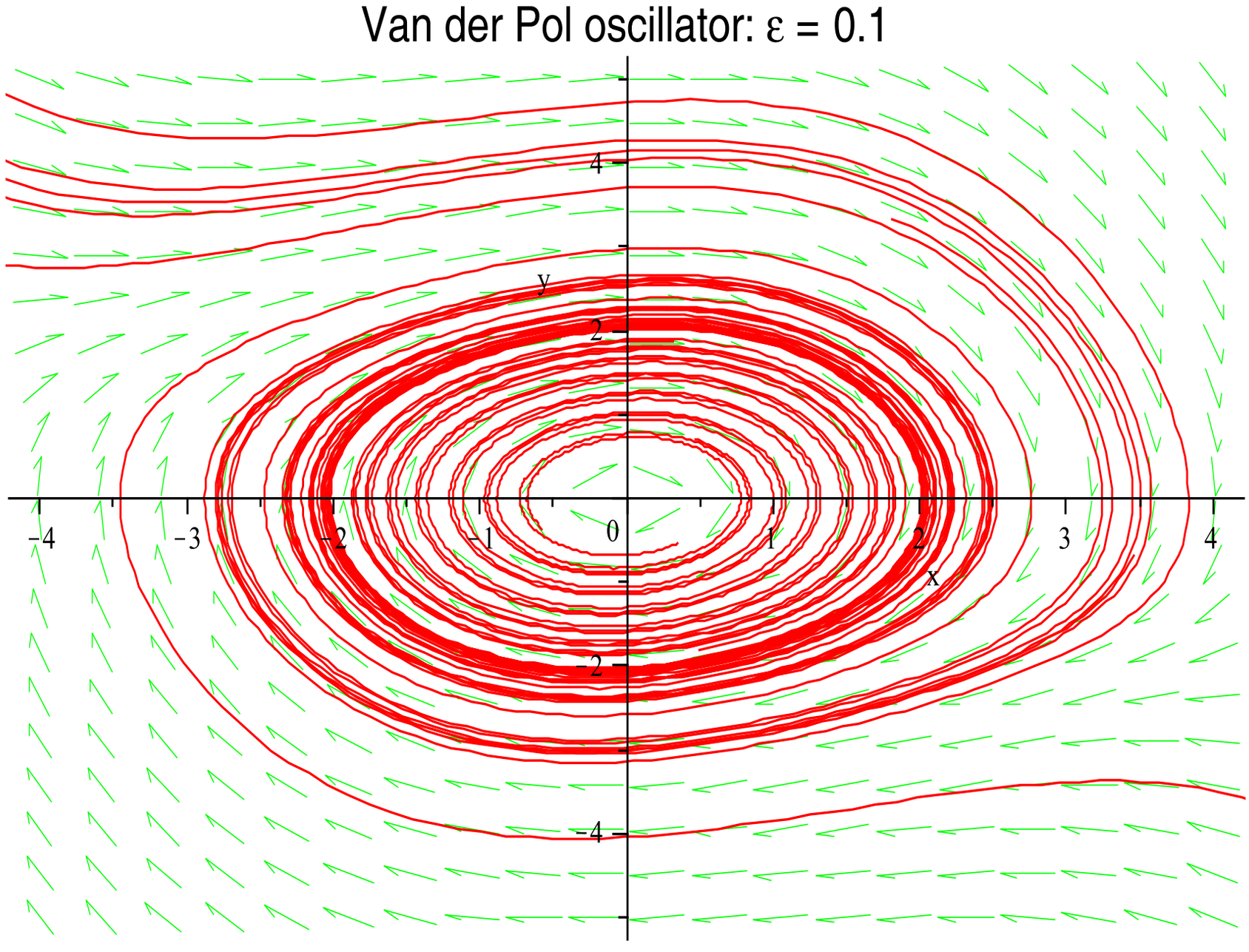
To convert this to a system of first order equations in two unknowns we let $y = x'$ and find

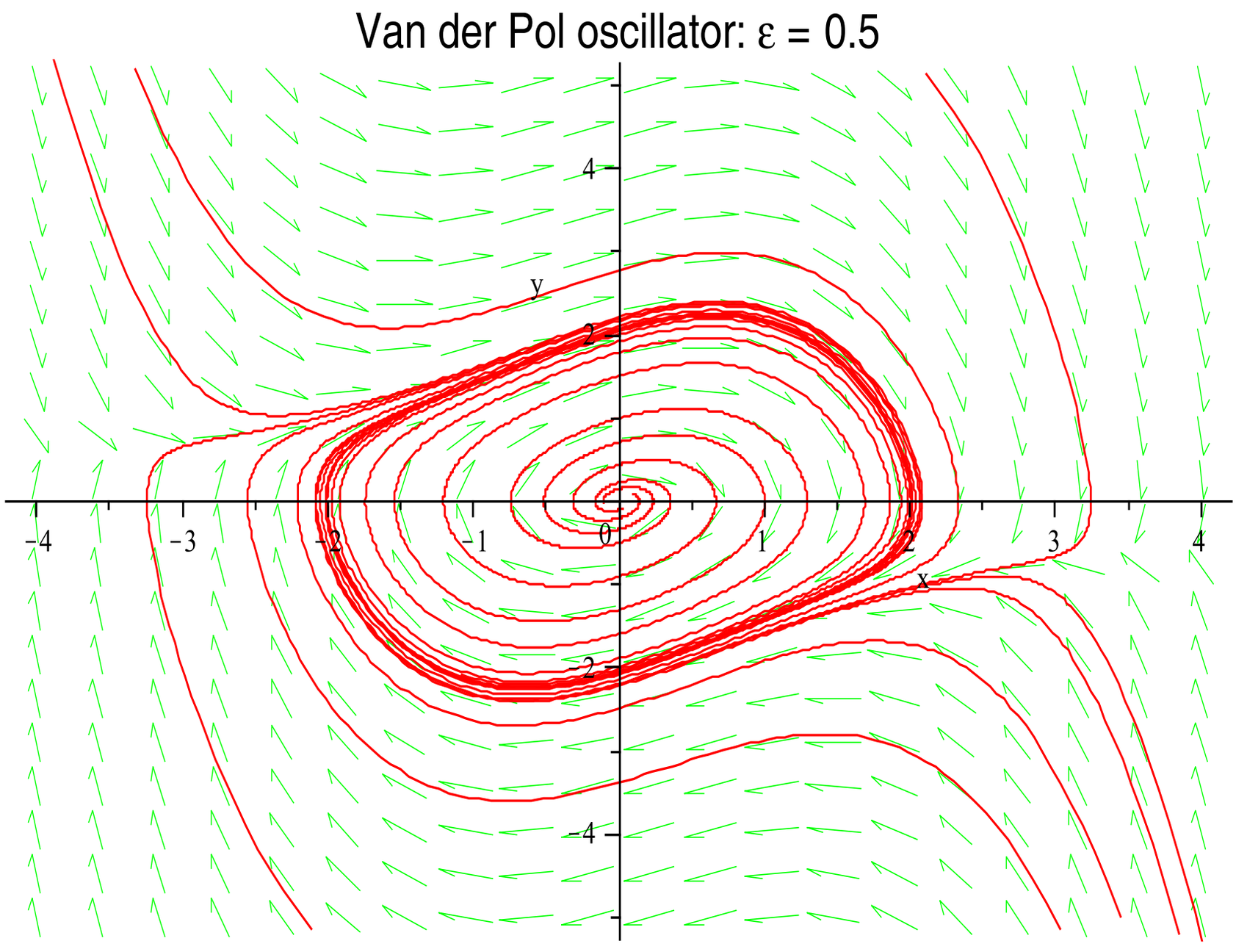
$$\mathbf{x}' = y, \qquad y' = -x + \epsilon(1 = x^2)y.$$

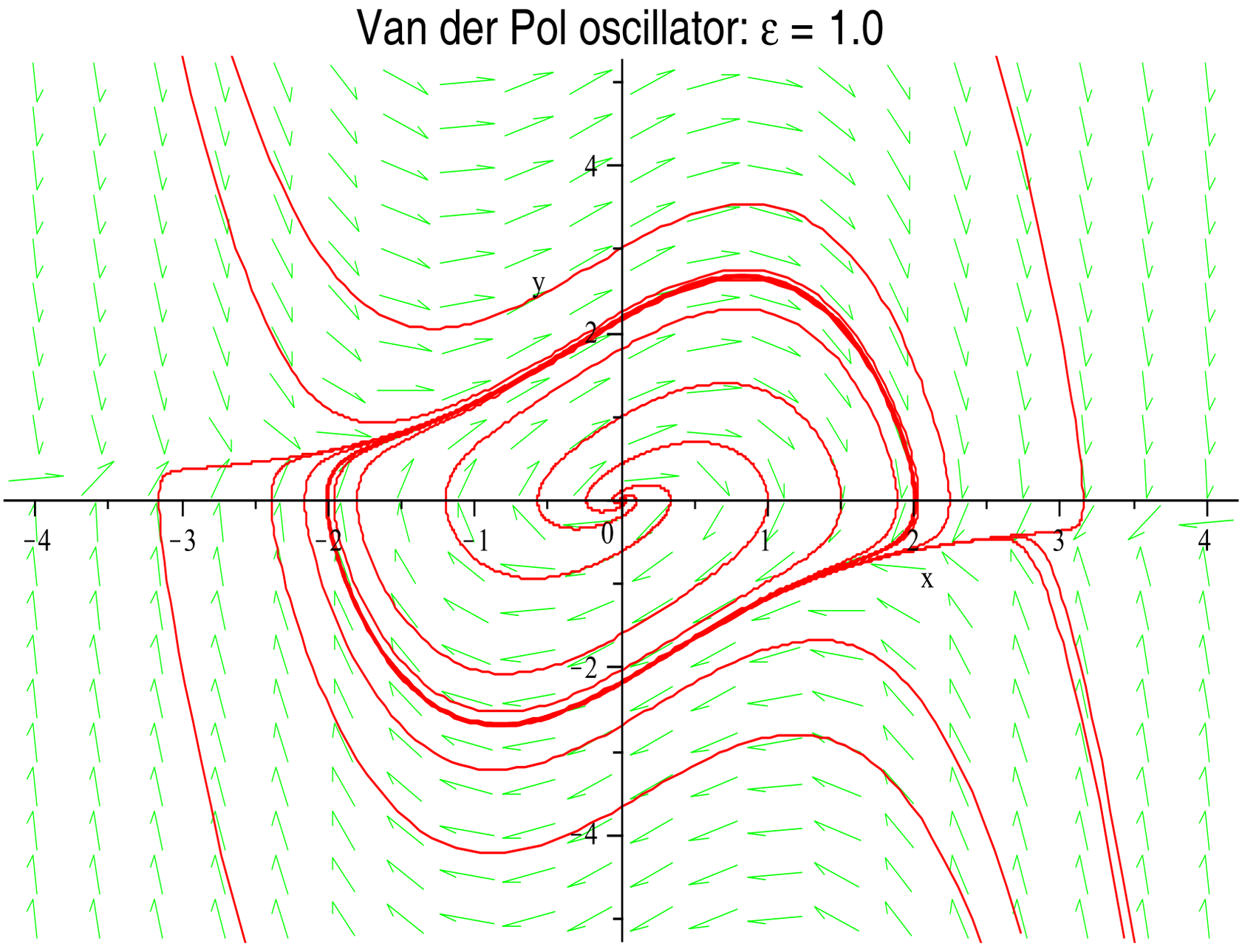
What follows are phase plane plots of this system for

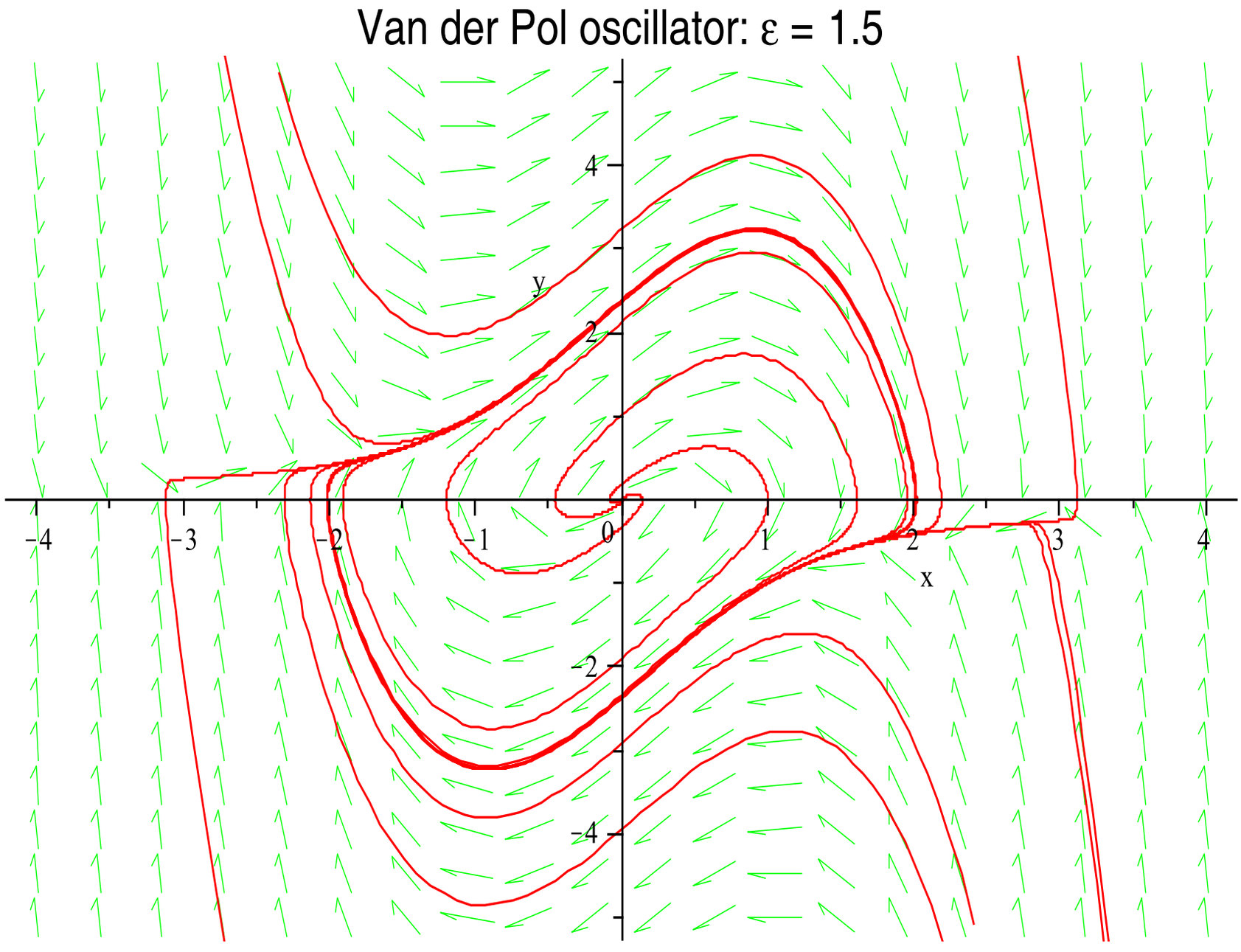
$$\epsilon = 0.1,\ 0.5,\ 1.0,\ 1,5,\ \text{and}\ 5.0.$$

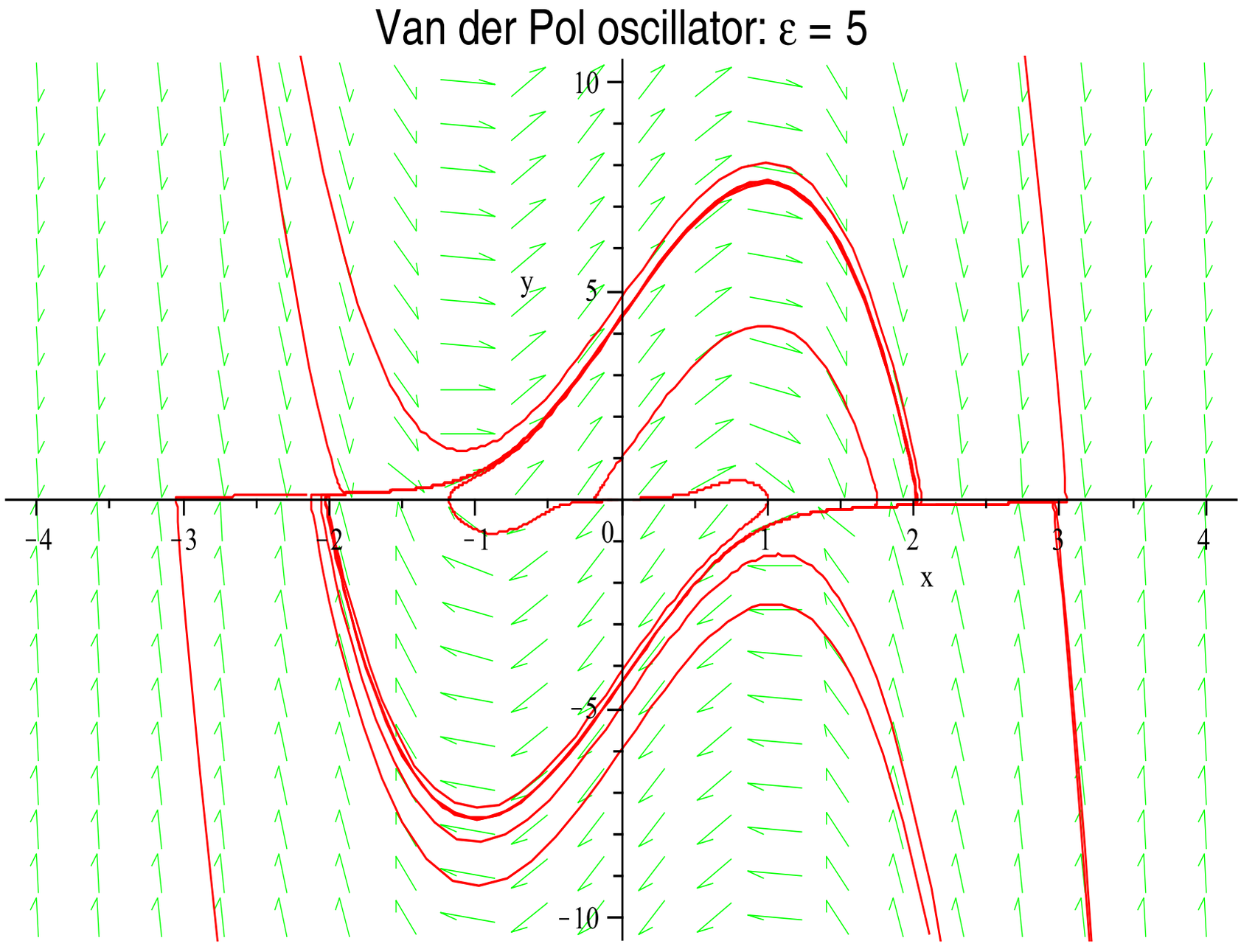
Poblem 6 of Section 7.5 in Greenberg has an interesting analysis of this sytem for large ϵ.


Van der Pol oscillator: ε = 0.1
y
x
4
2
−2
−4
−4
−3
−2
−1
0
1
2
3
4


Van der Pol oscillator: ε = 0.5
y
x
4
2
−2
−4
−4
−3
−2
−1
0
1
2
3
4


Van der Pol oscillator: ε = 1.0
y
x
4
2
−2
−4
−4
−3
−2
−1
0
1
2
3
4


Van der Pol oscillator: ε = 1.5
y
x
4
2
-2
-4
-4
-3
-2
-1
0
1
2
3
4


Van der Pol oscillator: ε = 5
10
5
y
−5
−10
−4
−3
−2
−1
0
1
2
3
4
x