The Van der Pol oscillator

The Van der Pol oscillator is governed by the second order equation

\[ x'' - \epsilon(1 - x^2)x' + x = 0. \]

To convert this to a system of first order equations in two unknowns we let \( y = x' \) and find

\[ x' = y, \quad y' = -x + \epsilon(1 - x^2)y. \]

What follows are phase plane plots of this system for

\[ \epsilon = 0.1, \ 0.5, \ 1.0, \ 1.5, \ \text{and} \ 5.0. \]

Problem 6 of Section 7.5 in Greenberg has an interesting analysis of this system for large \( \epsilon \).
Van der Pol oscillator: $\varepsilon = 0.1$
Van der Pol oscillator: \( \varepsilon = 0.5 \)
Van der Pol oscillator: $\varepsilon = 1.0$
Van der Pol oscillator: $\varepsilon = 1.5$
Van der Pol oscillator: $\varepsilon = 5$