Taylor series (with radii of convergence given):

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots = \sum_0^\infty x^n, \qquad |x| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_0^\infty \frac{x^n}{n!}, \qquad |x| < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_0^\infty \frac{(-1)^n x^{2n}}{(2n)!}, \qquad |x| < \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_0^\infty \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \qquad |x| < \infty$$

The Gamma function. For $x > 0$, $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\,dt$.

If x is not 0 or a negative integer, $\Gamma(x+1) = x\Gamma(x)$.

If n is a non-negative integer, $\Gamma(n+1) = n!$. $\Gamma(1/2) = \sqrt{\pi}$.

The Method of Frobenious—solution forms: $y_1(x) = x^r \sum_{n=0}^\infty a_n x^n$,

$$y_2(x) = y_1(x)(\ln x) + x^{r_1} \sum_{n=1}^\infty b_n x^n, \qquad y_2(x) = C y_1(x)(\ln x) + x^{r_2} \sum_{n=0}^\infty b_n x^n.$$

Bessel Functions.

A. The Bessel equation of order ν: $x^2 y'' + x y' + (x^2 - \nu^2) y = 0.$

B. If u solves the Bessel equation of order ν, and $b > 0$, then

$$y(x) = x^{\nu/\alpha} u\left(\alpha \sqrt{b} x^{1/\alpha}\right) \qquad \text{solves} \qquad y'' + \frac{a}{x} y' + b x^{c-a} y = 0,$$

where

$$\alpha = \frac{2}{c-a+2} \qquad \text{and} \qquad \nu = \frac{1-a}{c-a+2}.$$

C. Bessel functions:

$$J_\nu(x) = \left(\frac{x}{2}\right)^\nu \sum_{n=0}^\infty \frac{(-1)^k}{k!\Gamma(\nu+k+1)} \left(\frac{x}{2}\right)^{2k}$$

$$J_{-\nu}(x) = \left(\frac{x}{2}\right)^{-\nu} \sum_{n=0}^\infty \frac{(-1)^k}{k!\Gamma(k-\nu+1)} \left(\frac{x}{2}\right)^{2k}$$

$$Y_\nu(x) = \frac{(\cos \nu\pi) J_\nu(x) - J_{-\nu}(x)}{\sin \nu\pi}, \qquad \text{if } v \neq 0, 1, 2, \ldots.$$

$$Y_n(x) = \lim_{\nu \to n} Y_\nu(x), \qquad \text{if } n = 0, 1, 2, \ldots.$$

On the exam, the remaining part of this formula sheet will be Appendix C, the Laplace transform tables, from the text.