Since I was late posting this assignment, I amc changing the due date to Thursday. **Turn in starred problems Thursday 10/23/2007.**

Section 7.4: 2 (a), (b)*, (k)*

Section 7.4: 7

Section 7.5: 4*

Problem 7.A* Two interacting populations $x(t), y(t)$ are described by the equations

$$x' = (3 - x - y)x , \qquad y' = (2 - y)y .$$

(a) Find all the critical points of this system. You do not need to classify these.

(b) Sketch the first quadrant $x \geq 0$, $y \geq 0$ of the phase plane, indicating, by arrows or otherwise, regions where x and y are increasing, x is increasing and y decreasing, etc., and where the trajectories are horizontal and vertical.

(c) For each initial condition below, find (from your sketch or otherwise) $\lim_{t\to\infty} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$:

(i) $x(0) = 0$, $\quad y(0) = 3$;   (ii) $x(0) = 3$, $\quad y(0) = 3$;   (iii) $x(0) = 0$, $\quad y(0) = 0$.

In 7.4 (b), (k), please do the following:

- Find all singular points;
- Obtain the *linearized system* near each singular point and classify the origin of that system (as a saddle point, unstable or stable node, unstable or stable focus, or center). If the origin is a saddle point or a node, obtain the special straight-line trajectories. Sketch the phase plane of the linearized system.
- Discuss the nature of the singularity in the true system to the extent possible; in particular, discuss whether or not you can conclude that the singularity is of the same type as the linearized system. (See the discussion on page 362 of the Hartman-Grobman theorem).
- Use Maple (or some other program) to sketch enough of the phase plane to show all the singular points and the behavior of trajectories near them. Does this confirm your analysis above (it should)? Does it answer questions that you could not answer there?
- Here is Maple code for problem 2(b); you just have to enter it. It graphs six trajectories by giving six sets of inital conditions. You should find it fairly easy to modify this code to produce the plot for problem 2(k). You may want to change the ranges of the variables and to pick other initial points (you will probably need more than six trajectories to get a good picture of the behavior). Experiment until you get a nice plot.

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> with(DEtools);
> phaseportrait([diff(x(t),t)=1-y(t)^2,diff(y(t),t)=1-x(t)],
    [x(t),y(t)],t=-10..10,
    [[x(0)=1,y(0)=0],
     [x(0)=1,y(0)=0.75],
     [x(0)=1.5,y(0)=1],
     [x(0)=0.5,y(0)=1],
     [x(0)=1,y(0)=1.3],
     [x(0)=1,y(0)=-0.5]],
    x=-1..3, y=-2..2,stepsize=.05,linecolor=RED);
```