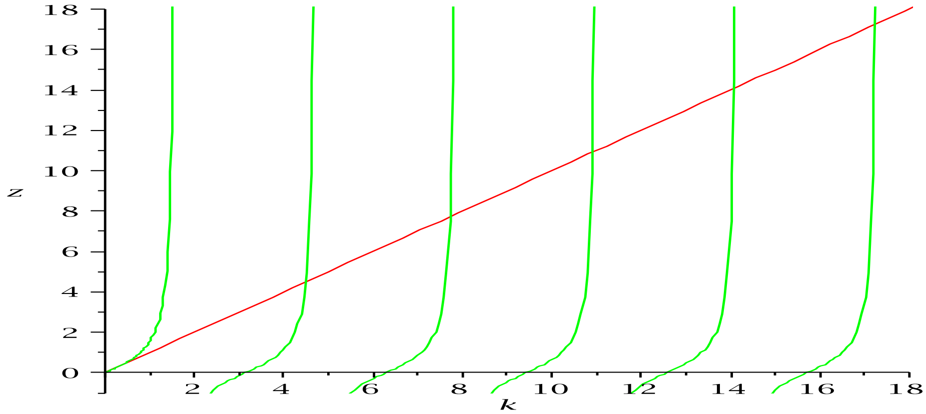
Eigenvalues and eigenfunctions of a Sturm-Liouville problem

In class on November 13 we discussed the problem

$$y'' + \lambda y = 0 \quad \text{for} \quad 0 < x < 1; \qquad y(0) = 0, \quad y'(1) - y(1) = 0.$$

We found the eigenvalues to be $\lambda_1 = 0$, $\lambda_n = \kappa_n^2$ for $n \geq 2$, where κ_n is the $(n-1)^{\text{st}}$ positive root of the equation $\kappa = \tan\kappa$. Graphically, the κ_n are determined as the intersections of the curves $z = \kappa$ and $z = \tan\kappa$:



Maple finds the first 6 eigenvalues as given in the table below; note that as expected, κ_n is very close to $(2n-1)\pi/2$ for n large:

n	κ_n	λ_n	$\phi_n(x)$
1		0	x
2	$4.493 = 0.954\,(3\pi/2)$	20.191	$\sin\kappa_n x$
3	$7.725 = 0.984\,(5\pi/2)$	59.680	$\sin\kappa_n x$
4	$10.904 = 0.992\,(7\pi/2)$	118.900	$\sin\kappa_n x$
5	$14.066 = 0.995\,(9\pi/2)$	197.858	$\sin\kappa_n x$
6	$17.221 = 0.997\,(11\pi/2)$	296.554	$\sin\kappa_n x$

Here are the first four eigenfunctions $\phi_1(x), \ldots, \phi_4(x)$; with a little good will one can believe that they indeed satisfy $\phi_n(1) = \phi_n'(1)$: