> ## The Fourier series of the step function
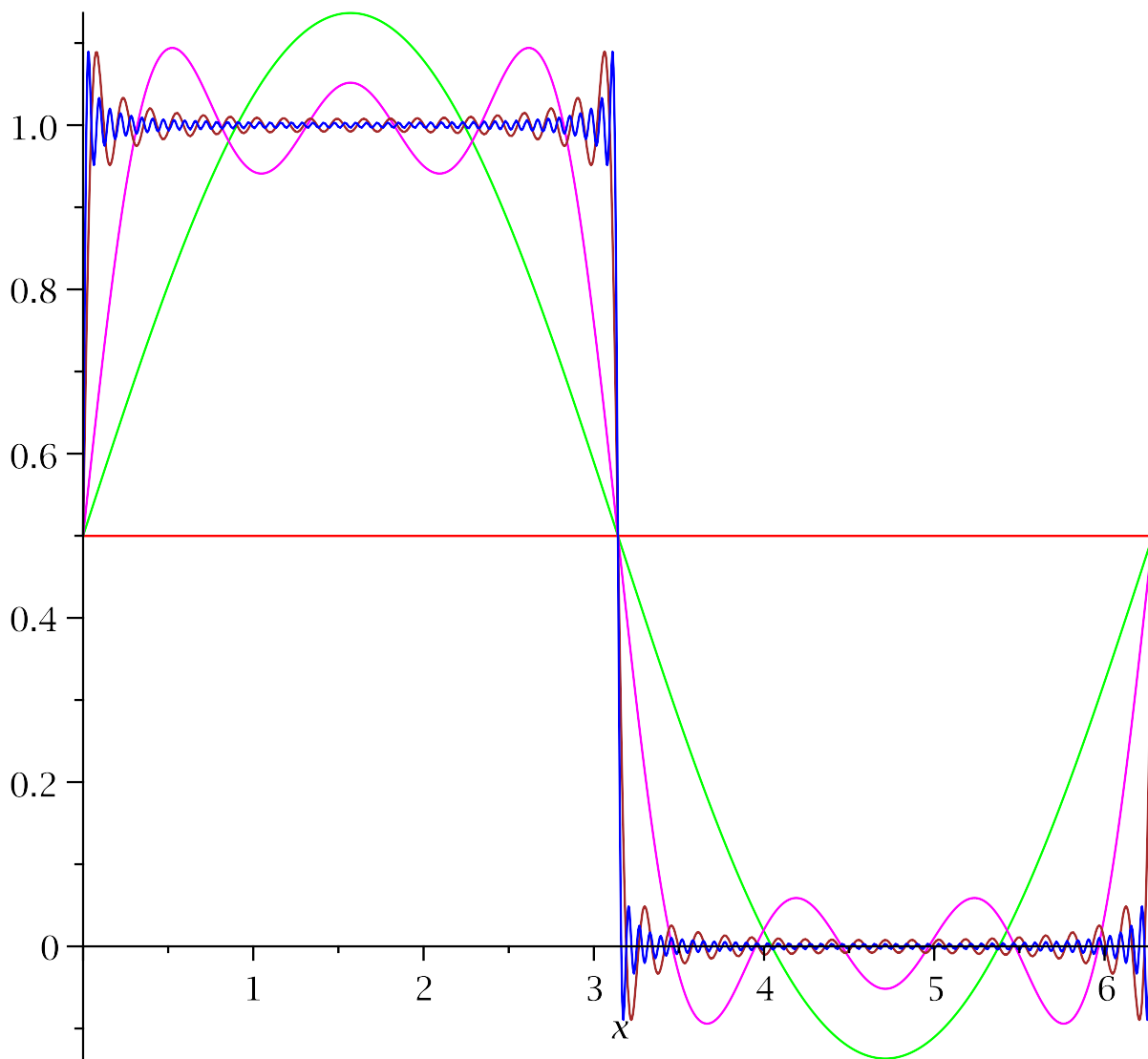f(x) = 1, 0 < x < Pi, f(x)=0, Pi < x < 2Pi

> `s := N -> 1/2+sum(2*sin((2*k+1)*x)/((2*k+1)*Pi),k=0..N-1);`

$$s := N \rightarrow \frac{1}{2} + \sum_{k=0}^{N-1} \frac{2\sin((2\,k+1)\,x)}{(2\,k+1)\,\pi} \tag{1}$$

> `s(3);`

$$\frac{1}{2} + \frac{2\sin(x)}{\pi} + \frac{2}{3}\frac{\sin(3\,x)}{\pi} + \frac{2}{5}\frac{\sin(5\,x)}{\pi} \tag{2}$$

> `plot([s(0),s(1),s(3),s(20),s(50)],x=0..2*Pi,color=[RED,GREEN,`
  `MAGENTA,BROWN,BLUE],thickness=1,numpoints=500);`

> **The Fourier series of the steady-state solution of**
$$y'' + 2y' + 10y = f(x),$$
    with f(x) as above.

```
> b := n -> (10-n^2)/(n^4-16*n^2+100): a := n -> -2*n/(n^4-16*
  n^2+100):
> y := N -> 1/20+2*sum((a(2*k+1)*cos((2*k+1)*x)+b(2*k+1)*sin((2*
  k+1)*x))/((2*k+1)*Pi),k=0..N-1);
```

$$y := N \rightarrow \frac{1}{20} + 2 \left(\sum_{k=0}^{N-1} \frac{a(2\,k+1)\,\cos((2\,k+1)\,x) + b(2\,k+1)\,\sin((2\,k+1)\,x)}{(2\,k+1)\,\pi} \right) \qquad (3)$$

```
> plot([y(1),y(2),y(20)],x=0..2*Pi,color=[RED,GREEN,BLUE],
  numpoints=500);
```