Multple-page homework must be STAPLED when handed in.

Turn in starred problems Tuesday 10/30/2007.

Section 7.4: 7 (a)*, (b), (c)
Section 7.5: 4*; 6
Section 9.6: 11; 12 (b), (c), (d)*, 13*

Problem 8.A* Two interacting populations $x(t), y(t)$ are described by the equations

$$x' = (3 - x - y)x\ , \qquad y' = (2 - y)y\ .$$

(a) Find all the critical points of this system. You do not need to classify these.

(b) Sketch the first quadrant $x \geq 0$, $y \geq 0$ of the phase plane, indicating, by arrows or otherwise, regions where x and y are increasing, x is increasing and y decreasing, etc., and where the trajectories are horizontal and vertical.

(c) For each initial condition below, find (from your sketch or otherwise) $\lim_{t\to\infty} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$:
(i) $x(0) = 0, \quad y(0) = 3;$ (ii) $x(0) = 3, \quad y(0) = 3;$ (iii) $x(0) = 0, \quad y(0) = 0.$

Comments: 1. I have listed problem 6 of Section 7.5 because it is a nice exercise if you want to find out more about the van der Pol oscillator, but it is certainly not required that you try it.

2. To clarify problem 13 of Section 9.6: we are considering solutions of n linear equations in n unknowns, that is, the equations have the form $A\mathbf{x} = \mathbf{b}$, where A is an $n \times n$ matrix, x is a column vector of n unknowns, and $\mathbf{b}$ is a given vector with n components. Here are the equations written out, but *it is much better to work with the matrix/vector notation:*

$$\begin{array}{ccccccccc} a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & = & b_2 \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ a_{n1}x_1 & + & a_{n2}x_2 & + & \cdots & + & a_{nn}x_n & = & b_n \end{array}$$

In (a) we assume that $\mathbf{b} = \mathbf{0}$; in part (b) $\mathbf{b}$ is arbitrary. To say that the set of solutions is a vector space is to say that a linear combination $\alpha\mathbf{x}^{(1)} + \beta\mathbf{x}^{(2)}$ of two solutions $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ is again a solution.