Multiple-page homework must be STAPLED when handed in.

Turn in starred problems Thursday 12/6/2007. Problems marked with two stars will be treated as extra credit.problems

Section 17.7: 8

Section 18.3: 10 (a), (b), (c)*, 14, 19*, 28**, 29*

12.A** Consider the following problem for the function $u(x,t)$:

$$9\,u_{xx} = u_t, \qquad 0 < x < 1, \quad t > 0; \tag{12.1}$$
$$u(0,t) = 0, \qquad \gamma u(1,t) + u_x(1,t) = 0, \qquad t > 0; \tag{12.2}$$
$$u(x,0) = 1, \qquad 0 < x < 1. \tag{12.3}$$

(a) Separate variables and investigate the eigenvalues of the resulting Sturm-Liouville problem. In particular, show that (i) if $\gamma > -1$ then all eigenvalues are positive, (ii) if $\gamma = -1$ then zero is an eigenvalue and all other eigenvalues are positive, and (iii) if $\gamma < -1$ then there is one negative eigenvalue and all other eigenvalues are positive. You will not be able to find the eigenvalues analytically. NOTE: We discussed the case $\gamma = -1$ in class, and Example 3 of Section 17.7 of our text is a model for the case $\gamma > -1$ (although in the text example the Robin boundary condition is imposed at the left, not the right, end of the rod).

(b) In each case above, find the solution of the problem as an infinite series. Express the coefficients as ratios of integrals, but do not attempt to evaluate them. The series and integrals will involve the eigenvalues from (a), so you won't be able to be too specific.

(c) Discuss the behavior of $u(x,t)$ as $t \to \infty$. You should find, in the various cases of (a), that (i) $u(x,t)$ approaches zero as $t \to \infty$; (ii) $u(x,t)$ approaches a non-zero steady state as $t \to \infty$; (iii) $u(x,t)$ becomes infinite ("blows up") as $t \to \infty$.

(d) What is the physical interpretation of the boundary condition at $x = L$ when $\gamma > 1$, and why, on physical grounds, does the solution blow up in that case?

12.B* Here is a variant of the periodic boundary condition problem of Section 17.8: Find the eigenvalues and eigenfunctions for

$$y'' + \lambda y = 0, \quad y(0) = -y(1), \quad y'(0) = -y'(0).$$

Find also the eigenfunction expansion of $f(x) = 1$.

Comments, hints, instructions: Problems 14 and 29 of Section 18.3 give two different approaches to the same problem. Only 29 is to be turned in, but you may find it useful to look at both of them.