640:495 Mathematical Finance; PROBLEMS Fall, 2006

16-19. Problems 1,2,3,4 on page 29 of Stamfli and Goodman.
20,21. Problems 1,2 on page 38 of Stampfli and Goodman.
22. In class, we show geometrically that in the one-period model there is no arbitrage if and only if $\ell < e^{r\tau} < g$, where g is the return on the stock if u occurs, ℓ is the return on the stock if d occurs, τ is the duration of the period, and r is the risk-free interest rate. This problem asks you to recast the proof by direct and algebraic reasoning as follows.

a). If $e^{r\tau} \leq \ell < g$, construct an arbitrage portfolio. Do the same if $\ell < g \leq e^{r\tau}$. This shows that no-arbitrage implies at least that $\ell < e^{r\tau} < g$.

b). Show algebraically that if $\ell < e^{r\tau} < g$, one cannot arbitrage using the stock and the risk-free interest rate.

23. Consider an underlying stock, and suppose that call and put options on that stock are both available at strikes K_1 and K_2, where $K_1 < K_2$, and expiration T. Assume that range forward contracts with range $[K_1, K_2]$ are also available for delivery at T. What, if anything, does the no-arbitrage assumption imply about the price of the range forward? (For range forwards, see the discussion in the lecture 2 notes.)
24. Three month calls on XYZ corporation stock at strike \$50 cost \$2, those at strike \$55 cost \$1.50. Three month puts on the stock at strike \$50 cost \$1.22, while those at strike \$55 cost \$3. The price of the stock today is \$49, and the nominal per annum risk free rate is %12. Is there an arbitrage opportunity? Why?
25. In class we derived put-call parity by comparing a portfolio long one call and short one put at the same strike X and expiration to a forward contract. Implicit in the no-arbitrage analysis was the assumption that we could obtain a forward contract at the same strike X. We can by-pass the use of the forward contract and argue put-call parity directly. This problem asks you do so as follows. Let the underlying be a stock whose price today is S_t and whose price at expiration at time T is S_T. Let r be the risk free interest rate. Let C_t and P_t denote the price today of the call and put options at strike X. Consider first the portfolio in which you borrow $S_t + P_t - C_t$ from the bank, buy a put, sell a call, and buy a share of stock. Derive an inequality from the no-arbitrage assumption. Derive the opposite inequality by the opposite portfolio, and combine the results to prove the put-call parity formula.

26–30. Problems 2–6 on page 36 of Stampfli and Goodman.