13.20. The null hypothesis can be rejected at the 0.05 and 0.10 levels since these levels are both larger than the \( p \)-value 0.0316.

13.21. The test statistic is \(|Z| = |(\bar{X} - 8)/(0.16/\sqrt{25})|\), and \(XZ\) is approximately standard normal assuming the null hypothesis. The data give a value \( z = 2.84\) for \( Z \). The \( p \)-value is the probability that a standard normal r.v. is larger than 2.84 in absolute value. Hence the \( p \)-value is \( P(|Z| \geq 2.84) = 2[1 - \Phi(2.34)] = 2[1 - 0.9977] = 0.0046.\)

13.25. In this solution we assume that the scores are normally distributed with unknown mean \( \mu \) and known standard deviation 8.6. An appropriate test statistic to test \( H_0 : \mu = 84.3 \) against \( H_1 : \mu > 84.3 \), is

\[
Z = \frac{\bar{X} - 84.3}{8.6/\sqrt{45}}
\]

This will be a standard normal random variable under the null hypothesis. This is a one-sided problem, so it is appropriate to use a rejection region of the form \( \{Z \geq k\} \). To get a test with a level of significance 0.01, we should choose \( k = z_{0.01} = 2.33 \). (This completes steps 1 and 2.)

Step 3 is to compute what \( Z \) is for the given data. This is

\[
\frac{87.8 - 84.3}{8.6/\sqrt{45}} = 2.73.
\]

Step 4: The value of the test statistic is greater than 2.33 and hence we reject the null hypothesis at the 0.01 significance level.

13.26 The \( p \)-value of the data in 13.25 is \( P(Z > 2.73) = 1 - \Phi(2.73) = 1 - 0.9968 = 0.0032 \). As this is less than 0.01, we reject the null hypothesis at the 0.01 level of significance.