Mathematics 373 **Numerical Analysis** **Spring 2004**

EXAMINATION I REVIEW PROBLEMS

No books
Show all of your work

1.(20 points) Suppose $g(x)$ is differentiable on [-1,4] and that $|g'(x)| < .2$ on [-1,4].

a) Show by example that $g(x)$ does not necessarily have a fixed point on the interval [-1,4].

b) Suppose we make the additional assumption that the function $g(x)$ in the problem statement is such that $g(-1)$ and $g(4)$ are in the interval [-1,4] . Explain how to compute a solution to g(x)=x by function iteration.

c) How many iterations of the method in b) are sufficient to compute a solution to $g(x) = x$ to error less than 10^{-6} ?

d) Suppose a zero of $g(x) - x$ is approximated by bisection. How many iterations of the bisection method on [-1,4] are sufficient to compute a solution to $g(x) = x$ to error less than 10^{-6} ?

2.(20 points) Suppose that f is a function passing through the data points (-1,1), (1,-5), (3,5), (4,1), and (5,31).

a) Compute an approximation to f(0) by finding a degree 4 interpolating polynomial passing through these data points.

b) Suppose that for all $n = 2, 3, 4, 5$ the n^{th} derivative satisfies $|f^{(n)}(x)| < n/10^4$ for $-1 \leq x \leq 5$. Give an upper bound for the error in your approximation to f(0).

3.(20 points) Use Newton's method to compute a root of $x^3 + 2x - 1 = 0$ to error less than .0005 .

4.(20 points)

A cube root of 17 in the interval $[2, 4]$ is computed by students A, B, C, and D by choosing among Newton's method, bisection, the secant method and the method of false position (in no particular order). The results each student obtained by her method are tabulated below:

A	B	C	D
3.000000000	3.087378640	3.087378640	2.582644628
2.500000000	2.892964801	2.487119247	2.571331512
2.750000000	2.791588955	2.571732227	2.571281591
2.625000000	2.728939942	2.571264156	2.571281590
2.562500000	2.686995895	2.571282262	
2.593750000	2.657591010	2.571281564	
2.578125000	2.636364849	2.571281591	

a) Identify the method that each student used to obtain her column of numbers, and explain your reasoning

b) Continue the method you have identified in each column to compute the next number each student's algorithm will produce, and extend each student's column.

c) Rank the methods by the speed of convergence to $\sqrt[3]{17}$ and explain your reasoning.

5.(20 points) The function $\sin(x)$ is graphed on the next page (solid line) for $-4.5 \leq x \leq 4.5$ and four points on the graph are boxed. Also graphed are the Taylor polynomial of degree 4 at one of these points, and the Lagrange, Hermite, free cubic spline and clamped cubic spline interpolating polynomials through the boxed points (also included is an expanded view of the graphs for $-4.5 \leq x \leq -.3$). Identify which of the graphs A, B, C, D or E correspond to these polynomials and explain your reasoning by describing the properties of these different versions of approximation by polynomials.

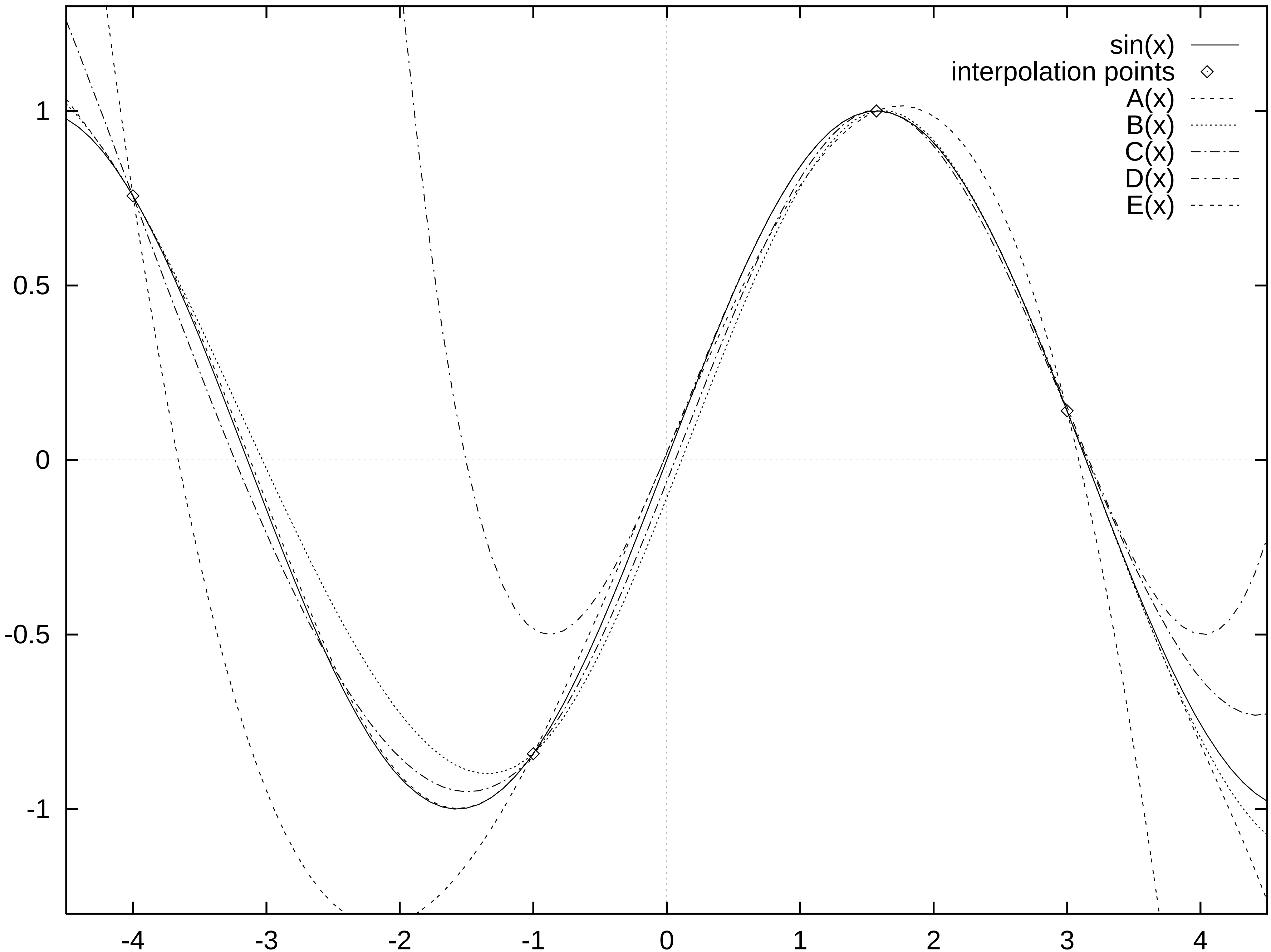
Taylor ______________

Lagrange ______________

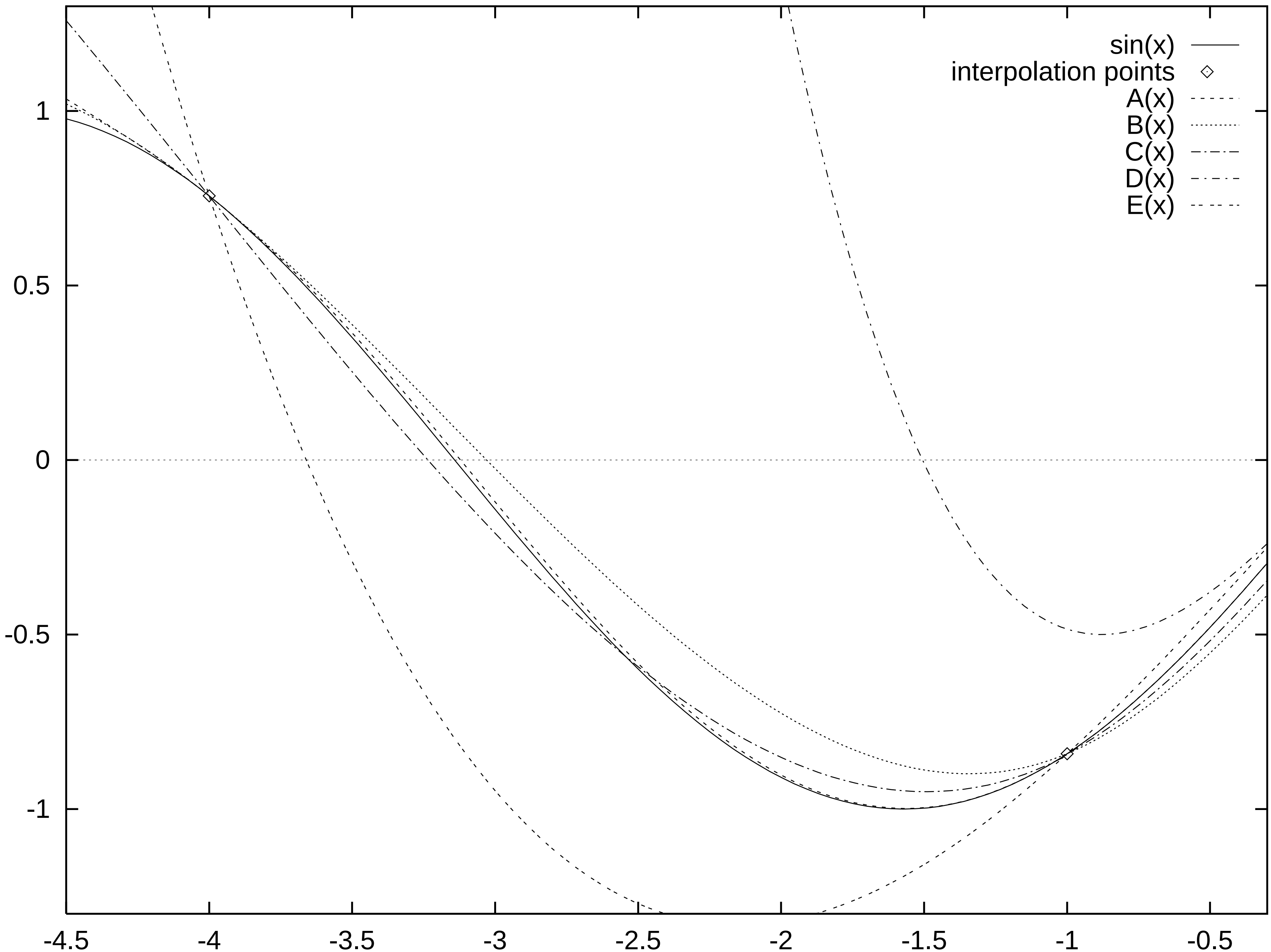
Hermite ______________

Free Spline ______________

Clamped Spline ______________



sin(x)
interpolation points
A(x)
B(x)
C(x)
D(x)
E(x)
1
0.5
0
-0.5
-1
-4
-3
-2
-1
0
1
2
3
4



sin(x)
interpolation points
A(x)
B(x)
C(x)
D(x)
E(x)
1
0.5
0
-0.5
-1
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5