Mathematics 373 Numerical Analysis Spring 2004

A short project on one of the topics below is due by the last day of classes, Monday, May 3. The project should consist of a well written description of the numerical problem to be solved followed by a description of the method used to solve it. Write in complete sentences, explaining how you chose the numerical method to use, and analyzing your computations. Include a description of any computer programs you use in your calculation. The project should not be more than 5 pages when written up. Turn in projects at class or at my office Hill 546 by May 3.

1. Find the arclength of the curve described by the function $y = e^x, 0 < x < 2$ to accuracy .001 . Explain carefully the method you choose to use and why your answer has the desired accuracy. Choose at least 2 methods from composite Simpson, Romberg integration, adaptive Quadrature and Gaussian Quadrature to compare. Plot carefully the curve for which you are computing the arclength.

2. The balance in a bank account in which interest is being earned at the rate of 5 percent compounded continuously and reinvested obeys the differential equation

   $$B' = .05B, B(0) = B_0.$$

   Suppose now that additional investments are made on a regular basis, but that larger deposits are made in certain months. The change in the bank balance could be modeled by

   $$B' = .05B + 2(\sin(2\pi t))^4, B(0) = B_0.$$

   Take an initial deposit of 1000 and use a fourth order Runge-Kutta method to compute the balance after 2 years for each of these differential equations.

3. Consider the differential equation

   $$y' = x + y, y(0) = 2.$$

   For $0 \leq x \leq 1$ compare the predictor–corrector method of solution to what happens if you use the Adams-Bashforth method without correction. Which is the more accurate method?

4. Solve the system

   $$C_1' = -.2C_1, C_1(0) = .3$$

   $$C_2' = -.4(C_2 - C_1), C_2(0) = 0$$

   numerically for $0 \leq t \leq 10$. Compare Euler's method and the midpoint method for this system of differential equations that describe the concentration of dye in two tanks. Plot the solutions to the system that you obtain.