March 9, 1999 **Math 373** Quiz 3

Name: ANSWERS last 4 SSN:

Your technician has obtained the following data about a function $H(x)$:

$$\begin{aligned} H(1.5000) &= .93160 \\ H(1.5250) &= .95522 \\ H(1.5500) &= .97942 \end{aligned}$$

1. Your boss needs the value of $H'(1.5000)$. Use Richardson extrapolation and the formula

$$H'(x_o) = \frac{H(x_o+h) - H(x_o)}{h} + O(h)$$

to obtain an $O(h^2)$ approximation to $H'(1.5000)$.

$R_{1,1} = H'(h=.05) = \frac{.97942-.93160}{.05} = .9564 + O(h)$

$R_{2,1} = H'(h=.025) = \frac{.95522-.93160}{.025} = .9448 + O(h)$

$R_{2,2}(h=.05) = \frac{2R_{2,1}-R_{1,1}}{1} = 2\cdot(.9448) - .9564 = .9332 + O(h^2)$

2. To get an $O(h^3)$ approximation to $H'(1.5000)$ you should find out data about $H(u)$ for what number u?

You do not wish to make the value of h larger than the .05 as that will cancel out part of the savings from going to a higher order approximation. So you should plan to add an R_{31} by finding out the value of $H(1.5125)$.

3. The technician makes the necessary measurements to ascertain that $H(u) = .94334$ for the u in 2. What value should you give your boss for $H'(1.5000)$.

$R_{3,1} = \frac{.94334-.93160}{.0125} = .9392$ $R_{3,2} = 2R_{3,1}-R_{2,1} = 2\cdot(.9392)-.9448 = .9336$ $R_{3,3} = \frac{4R_{3,2}-R_{2,2}}{3} = \frac{4\cdot(.9336)-.9332}{3} = .9337$

You should report to your boss that $H'(1.5000) \approx .9337$.

4. Would adding more measurements of data points help? Why or why not.

Adding another point would probably not help because of round off error. You start with 5 significant digits and even in the first $O(h)$ approximation you lose one of them. Your values of R_{23} and R_{32} differ from R_{33} in the fourth decimal place. So this fourth decimal place is the least significant digit which is somewhat in doubt due to round off error. More round off errors will not help this.