Math 373 — Spring 2000

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Answers to Homework 5 (lecture 5 – Due 2/8/00)

The odd numbered exercises have answers in the book, so you can check your work. Only a few will be answered here. Even numbered ones are answered here.

Exercise 1 (Page 75: 2) *Let $f(x) = -x^3 - \cos(x)$ and $p_0 = -1$. Use Newton's method to find p_2. Could $p_0 = 0$ be used?*

Set $g(x) = x - \frac{f(x)}{f'(x)}$. Let $p_0 = -1$ and $p_1 = g(-1) = -.8803329$, $p_2 = g(-.8803329) = -.86568416$, $p_3 = g(-.86568416) = -.86547408$, $p_4 = g(-.86547408) = -.86547403$ and that is about as good as we can realistically expect to do without changing the precision of the computations.

If we start out with $p_0 = 0$, our first iteration of Newton's method will get us a divide by zero error at a place that is not a zero of f, so we cannot even modify Newton to work for it.

Exercise 2 (Page 75: 6a, 8a) *Use Newton, Secant, and False Position to find a zero of*

$$f(x) = e^x + 2^{-x} + 2\cos(x) - 6 = 0$$

to within 10^{-5} on the interval $[1, 2]$.

We do False Position first because in the process we can check that there is a root in the given interval. $p_0 = 1$, $p_1 = 2$, $q_0 = f(p_0) = -1.7011136$, $q_1 = f(2) = .80676243$ so by the intermediate value theorem we indeed straddle a root. $p_2 = p_1 - q_1 \frac{p_1 - p_0}{q_1 - q_0} = 2 - .80676243 \cdot \frac{2-1}{.80676243 - (-1.7011136)} = 1.6783085$, $q_2 = f(1.6783085) = -.54567379$. The root is between p_2 and p_1.

$p_3 = p_2 - q_2 \frac{p_2 - p_1}{q_2 - q_1} = 1.6783085 - (-.54567379) \frac{1.6783085 - 2}{-.54567379 - (.80676243)} = 1.8081029$. $q_3 = f(p_3) = -8.5738595 \times 10^{-2}$. The root is between p_3 and p_1.

(Here is the first place we differ from the secant method.) $p_4 = p_3 - q_3 \frac{p_3 - p_1}{q_3 - q_1} = 1.8081029 - (-8.5738595 \times 10^{-2}) \frac{1.8081029 - 2}{-8.5738595 \times 10^{-2} - .80676243} = 1.8265376$, $q_4 = f(p_4) = -1.1645057 \times 10^{-2}$ so the root is between p_4 and p_1. $p_5 = p_4 - q_4 \frac{p_4 - p_1}{q_4 - q_1} = 1.8265376 - (-1.1645057 \times 10^{-2}) \frac{1.8265376 - 2}{-1.1645057 \times 10^{-2} - .80676243} = 1.8290058$, $q_5 = f(1.8290058) = -1.549064 \times 10^{-3}$ so the root is between p_5 and p_1. Continuing:

$p_6 = 1.8290058 - (-1.549064 \times 10^{-3}) \frac{1.8290058 - 2}{-1.549064 \times 10^{-3} - .80676243} = 1.8293335$, $q_6 = f(1.8293335) = -2.05484 \times 10^{-4}$.

$p_7 = 1.8293335 - (-2.05484 \times 10^{-4}) \frac{1.8293335 - 2}{-2.05484 \times 10^{-4} - .80676243} = 1.829377$, $q_7 = f(1.829377) = -2.7077 \times 10^{-5}$.

$p_8 = 1.829377 - (-2.7077 \times 10^{-5}) \frac{1.829377 - 2}{-2.7077 \times 10^{-5} - .80676243} = 1.8293827$, $q_8 = f(1.829377) = -2.7077 \times 10^{-5}$.

$p_9 = 1.8293827 - (-2.7077 \times 10^{-5}) \frac{1.8293827-2}{-2.7077\times10^{-5}-.80676243} = 1.8293884$, $q_9 = f(1.8293884) = .00001968$. The root is between p_8 and p_9;and they differ by less than 10^{-5}. Return the approximate root 1.8293884. We are certainly within the required error estimate.

For Newton, set $g(x) = x - \frac{f(x)}{f'(x)}$ and start with $p_0 = 2$. $p_1 = g(2) = 1.8505213$, and $p_2 = g(1.8505213) = 1.8297512$, $p_3 = g(1.8297512) = 1.8293837$, $p_4 = g(1.8293837) = 1.8293836$ and the only difference is in the least significant digit so you will not get any closer without changing precision. Return the approximate root 1.8293836.

For secant, begin as with false position. $p_0 = 1$, $p_1 = 2$, $q_0 = f(p_0) = -1.7011136$, $q_1 = f(2) = .80676243$. $p_2 = p_1 - q_1 \frac{p_1-p_0}{q_1-q_0} = 2 - .80676243 \cdot \frac{2-1}{.80676243-(-1.7011136)} = 1.6783085$, $q_2 = f(1.6783085) = -.54567379$.

$p_3 = p_2 - q_2 \frac{p_2-p_1}{q_2-q_1} = 1.6783085 - (-.54567379) \frac{1.6783085-2}{-.54567379-(.80676243)} = 1.8081029$. $q_3 = f(p_3) = -8.5738595 \times 10^{-2}$.

$p_4 = p_3 - q_3 \frac{p_3-p_2}{q_3-q_2} = 1.8081029 - (-8.5738595 \times 10^{-2}) \frac{1.8081029-1.6783085}{-8.5738595\times10^{-2}-(-.54567379)} = 1.8322985$, $q_4 = f(1.8322985) = 1.1984702 \times 10^{-2}$.

$p_5 = 1.8322985 - 1.1984702\times10^{-2} \frac{1.8322985-1.8081029}{1.1984702\times10^{-2}-(-8.5738595\times10^{-2})} = 1.8293312$, $q_5 = f(1.8293312) = -2.14917 \times 10^{-4}$

$p_6 = 1.8293312 - (-2.14917 \times 10^{-4}) \frac{1.8293312-1.8322985}{-2.14917\times10^{-4}-1.1984702\times10^{-2}} = 1.8293835$, $q_6 = f(1.8293835) = -4.18 \times 10^{-7}$.

$p_7 = 1.8293835 - (-4.18 \times 10^{-7}) \frac{1.8293835-1.8293312}{-4.18\times10^{-7}-(-2.14917\times10^{-4})} = 1.8293836$. As with Newton's method, we stop here because the only difference between two iterates is in the least significant digit and we would have to increase precision to get any better. (Actually, I am using two more digits than I am printing out.) We return the approximate root 1.8293836.