Workshop 9, Math 311

1. Draw the graph of each of the following functions. For each case, determine whether the function is continous or discontinuous at $x = 0$ and prove your claim.

a) $f(x) = \{ x^2 \quad \text{if } x \neq 0$

b) $g(x) = \begin{cases} x^2 - 1 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

c) $k(x) = \begin{cases} \frac{x^2}{x(x+1)} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

d) $f(x) = \begin{cases} 0 & \text{if } x \neq 1/n \text{ for all } n \in \mathbf{N} \\ 1 & \text{if } x = 1/n \text{ for some } n \in \mathbf{N} \end{cases}$

2. Let $f : \mathbf{R} \to \mathbf{R}$ and $c \in \mathbf{R}$. We say that f is *continuous at c from the left* if for every sequence of real numbers (x_n) such that $x_n < c$ for all n then

$$\lim_{n\to\infty} x_n = c \quad \text{implies} \quad \lim_{n\to\infty} f(x_n) = f(c)$$

.

a. Write the analogous definition for *continuous at c from the right.*

b. Show that f is continuous at c if and only if f is continuous at c from both the right and the left.

c. Use your result in b. to prove that the following function is continuous on all of $\mathbf{R}$.

$$h(x) = \begin{cases} x^2 \cos(x) & \text{if } x \geq 0 \\ \sin(x) & \text{if } x < 0 \end{cases}$$