Workshop 8, Math 311

1. In each of the cases below, f is a function whose domain is $\mathbf{R}$ and whose range is the set of two points $\{0, 1\}$. For each case, answer the following questions and prove your assertions:

i) For which $a \in \mathbf{R}$ does $\lim_{x \to a} f$ exist?
ii) If this limit exists, what is it?

a) $f(x) = \begin{cases} 0 & \text{if } x \neq 1/n \text{ for all } n \in \mathbf{N} \\ 1 & \text{if } x = 1/n \text{ for some } n \in \mathbf{N} \end{cases}$

b) $f(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{all other } x \end{cases}$

c) $f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$

d) $f(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

2. Determine the set of cluster points for each of the following sets. Which cluster points belong to the sets?

a. $A = \{x_n \ : \ n \in \mathbf{N}\}$ where the sequence (x_n) converges to some number c.

b. $M = I \cap \mathbf{Q}$ where $I = [0, 1]$.

3. Show that $\lim_{x \to c} x^n = c^n$ for $c \in \mathbf{R}$ and $n \in \mathbf{N}$.