Workshop 5, Math 311

1. Let (a_n) be a sequence that satisfies:

$$\dagger \qquad \text{For any } n \in \mathbf{N} \text{ there exists an integer } m \text{ with } m > n \text{ such that } a_m \cdot a_n < 0.$$

a. Find examples of convergent and divergent sequences that satisfy $\dagger$.

b. Suppose that (a_n) converges and that its limit is a. What can you say about a? Prove your assertion.

2. Let (b_n) be a sequence that satisfies:

$$* \qquad \text{For any } n \in \mathbf{N} \text{ there exists an integer } m \text{ with } m > n \text{ such that } b_m > b_n.$$

a. Find examples of convergent and divergent sequences that satisfy $*$.

b. Does it follow from $*$ that $b_m > b_n$ for all $n, m \in \mathbf{N}$? If not, give an example. If so, prove your assertion.

c. Suppose that (b_n) converges and that its limit is b. What is the relation of b to any of the elements b_n as a result of $*$?

3. a. By doing a few sample calculations, determine for which n the following inequality holds:

$$0 < 2^n/n! < 2(2/3)^{n-2}$$

b. Prove your assertion.

c. Use b. to show that $\lim(2^n/n!) = 0$.

d. Can you come up (and prove) a similar result for the sequence $(k^n/n!)$ where $k \in \mathbf{R}$?