Workshop 4, Math 311

1. Consider each of the following statements. If a statement is true, then prove it. If a statement is false, then give a counterexample and "fix" the statement so that it is true. In addition, prove the fixed statement.

a. If $\lim_{n\to\infty} x_n = x$ and if $a < x_n < b$ for all n, then $a < x < b$.

b. If $|x_n|$ converges to 0, then x_n converges to 0.

2. Consider the sequence whose nth term is

$$x_n = \frac{1}{n(n+1)(n+2)}.$$

a. Using the definition of convergence, prove that $x_n \to 0$.

b. For $\epsilon = 1, .1, .01, .001$, what is $K(\epsilon)$? (Choose $K(\epsilon) \in \mathbf{N}$).

c. Is $K(\epsilon)$ the smallest number n that satisfies

$$\frac{1}{n(n+1)(n+2)} < \epsilon \ ?$$

If not, what is that smallest number?

d. When proving whether (x_n) converges, is it important that $K(\epsilon)$ be "as small as possible"? Why or why not?

3. Use the definition of convergence to establish the following limits.

a. $\lim(\frac{1}{\ln(n+1)}) = 0$

b. $\lim(\frac{n^2-1}{n^2+1}) = 1$

4. Use the definition of convergence to establish that the following sequences x_n do not converge to the given number x.

a. $x_n = 1 + (-1)^n$ and $x = 0$

b. $x_n = 1/n$ and $x = 1/2$