# Workshop 2, Math 311

1. Consider the set of polynomials $R[x]$ with real coefficients with the usual addition and multiplication operations.

(a) Which of the rules (A1) - (D) hold for $R[x]$?

Now consider the following order relation on $R[x]$. We say that a polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$$

is *positive* if $a_n > 0$.

(b) Show that $R[x]$ satisfies the Trichotomy Property.

(c) Is $R[x]$ Archimedean? (Hint: Compare constant polynomials with polynomials of degree 2, degree 3, etc. What are the order relations between them? )

2. Let $S$ be a nonempty bounded subset of $R$ and let $a \in R$.

(a) (warm-up problem) Prove that $\sup(a + S) = a + \sup S$.

(b) (more warm-up) Now let $a > 0$. Let $aS := \{as \mid s \in S\}$. Prove that $\sup(aS) = a\sup S$.

Let $A$ and $B$ be nonempty bounded subsets of the positive real numbers. Define

$$A \cdot B := \{ab \mid a \in A \;\; b \in B\}.$$

(c) Show that $\sup(A \cdot B) = (\sup A)(\sup B)$.

(d) Does this equality still hold if we don't assume that $A$ and $B$ are positive?

3. Find examples of non-empty bounded sets $S$ and $T$ such that all of the following conditions hold:

$$\sup S = 1 \text{ and } \sup T = 1 \text{ and } \inf S = 0 \text{ and } \inf T = 1 \text{ and } S \cap T = \emptyset.$$

4. Let $V_\epsilon(a)$ and $V_\delta(b)$ be neighborhoods of the real numbers $a$ and $b$.

(a) Find conditions on $a, b, \epsilon$ and $\delta$ so that $V_\epsilon(a) \cap V_\delta(b) = V_\gamma(c)$ for some $\gamma > 0$ and some number $c$.

(b) Find conditions on $a, b, \epsilon$ and $\delta$ so that $V_\epsilon(a) \cup V_\delta(b) = V_\gamma(c)$ for some $\gamma > 0$ and some number $c$.

(Hint: You already worked out the case $a = b$ in your homework.)