Workshop 1, Math 311

1. Can there be a smallest positive real number?

(a) Prove that if $a > 0$, then $1/a > 0$.

(b) By comparing a and $(a)(1/2)$, prove the following statement: *If* $a \in R$ *is such that* $0 \leq a < \epsilon$ *for every* $\epsilon > 0$*, then* $a = 0$.

(c) Prove that if $a, b \in R$ and if, for every $\epsilon > 0$ we have $a \leq b + \epsilon$, then $a \leq b$.

2. Let n be a natural number.

(a) Prove the following by induction:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

(b) Prove a similar formula for the sum of all *odd* numbers from 1 to n (you might need to consider the cases when n is odd and when n is even).

3. Let $0 < a < b$ and $0 < c < d$. Prove that $0 < ac < bd$.

4. We say that a property holds for "all n sufficiently large" if there exists a number n_0 such that the property holds for all natural numbers $n \geq n_0$.

(a) For which n do we have $n^3 \leq n!$? In other words, what is "sufficiently large" in this case?

(b) Prove that $n^3 \leq n!$ for all n sufficiently large. (You might need to use 3).

(c) Prove that $n^6 \leq n!$ for all n sufficiently large.

(d) Let k be a fixed natural number. Prove that $n^k \leq n!$ for all n sufficiently large.