Advanced Calculus I 311

First Midterm

Instructions: Answer the following questions in your blue books. Your solution to each problem must appear on consecutive pages. Write the corresponding problem number at the top of each page.

(10 pts) 1. Prove that $(-1)(-1) = 1$.

(15 pts) 2. Prove that $\sqrt{5}$ is an irrational number.

(15 pts) 3. Find all $x \in \mathbf{R}$ such that $|2x + 1| = 5 - |x|$.

(25 pts) 4. Let A and B be bounded subsets of $\mathbf{R}$ such that

$$a \leq b \ \forall a \in A, \ \forall b \in B. \qquad (*)$$

   a. Prove that $\sup A \leq b$ for every $b \in B$.
   b. Use a. to prove that $\sup A \leq \inf B$.
   c. By finding a counterexample, show that b. doesn't necessarily hold if $(*)$ is not true.

(35 pts) 5. Let $S = \{(-1)^n/n \mid n \in \mathbf{N}\}$.
   a. Write out the elements of S corresponding to $1 \leq n \leq 10$. Draw them on a number line.
   b. What do you think that $\sup S$ and $\inf S$ are? Prove your claim for $\sup S$.
   c. Prove that for all $\epsilon > 0$, the neighborhood $V_\epsilon(0)$ contains an element of S.
   d. Prove that for each element $(-1)^n/n \in S$, there exists an $\epsilon > 0$ such that

$$V_\epsilon((-1)^n/n) \ \cap \ S = \{(-1)^n/n\}$$

(in other words, prove that there exists an ϵ-neighborhood of $(-1)^n/n$ which contains no other element of S besides $(-1)^n/n$).