1. (a) Every column of A is a pivot column, so if $A\mathbf{x} = \mathbf{0}$ then $\mathbf{x} = \mathbf{0}$.

(b) rref(A) is the identity matrix, so A has pivot in every column. Hence S is independent.

2. (b) Yes: A has a pivot in every row, so $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b} \in \mathbb{R}^3$.

3. Now $\text{rref}(A) = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, so S dependent (no pivot in column #3) and S does not span $\mathbb{R}^3$ (no pivot in row #3).

4. (a) Inconsistent (b) $\mathbf{x} = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 4 \end{bmatrix}$ (c) Three parameters (free variables). General solution

$$\mathbf{x} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 0 \\ 3 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 2 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}$$

5. (a) $R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ (b) Impossible. (c) Impossible. (d) $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(e) Impossible: $\mathbf{x} = \mathbf{0}$ is always a solution. (f) $R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (g) Impossible.

6. If $A\mathbf{u} = \mathbf{b}$ and $A\mathbf{v} = \mathbf{b}$, then $A(c\mathbf{u}+d\mathbf{v}) = cA\mathbf{u}+dA\mathbf{v} = (c+d)\mathbf{b}$. If $\mathbf{b} = \mathbf{0}$, then $A(c\mathbf{u}+d\mathbf{v}) = \mathbf{0}$. If $\mathbf{b} \neq \mathbf{0}$, then $(c+d)\mathbf{b} \neq \mathbf{b}$ when $c + d \neq 1$.

7. (b) $AB = \begin{bmatrix} 0 & 8 & -1 \\ -5 & 2 & 1 \end{bmatrix}$ (c) $3C - 2B^T = \begin{bmatrix} -12 & 11 \\ 5 & -13 \\ 14 & 3 \end{bmatrix}$ (d) $BC = \begin{bmatrix} -4 & 2 \\ 8 & -9 \end{bmatrix}$

(e) $CAB = \begin{bmatrix} -15 & -10 & 5 \\ 15 & 18 & -6 \\ -5 & 34 & -3 \end{bmatrix}$ (g) $C^TC = \begin{bmatrix} 29 & -11 \\ -11 & 19 \end{bmatrix}$

8. (a) $r = n$ and $n < m$. (b) $r = m$ and $m < n$. (c) $r = m$ and $m = n$.
(d) $r < n$ and $r < m$. (e) $r = n$ and $n \leq m$.

9. (i) $E = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = E^{-1}$ (ii) $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(iii) $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $E^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/7 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

10. (a) $3\mathbf{a}_1 - 2\mathbf{a}_2 + \mathbf{a}_3 = \mathbf{0}$. (b) $\mathbf{a}_3 = -3\mathbf{a}_1 + 2\mathbf{a}_2 = \begin{bmatrix} 2 \\ -8 \\ -9 \end{bmatrix}$.

11. (b) Set $C = B^{-1}A^{-1}$. Then $ABC = I = CAB$ (associative law), so $C = (AB)^{-1}$.

(c) Set $D = (A^{-1})^T$. Then $A^TD = (D^TA)^T = I$ (transpose reverses multiplication). Likewise $DA^T = I$. Hence $D = (A^T)^{-1}$.

12. Inverse matrix is $\begin{bmatrix} -1 & -5 & 3 \\ 1 & 2 & -1 \\ 1 & 4 & -2 \end{bmatrix}$