

Integration Techniques and Memorization

On page 383 of the textbook you can see a table of 19 integration formulas. On the previous page there is a marginal note explaining how an integral of the form $\int \tan^m x \sec^n x dx$ can be evaluated when m is even and n is odd. This marginal note states that you can use formula (14) on the table on page 383. Since we do not allow formula sheets during exams, a student may conclude that it is useful to memorize all 19 formulas.

Math 152 students DO have to memorize MANY formulas before taking an exam. We expect you to know the Quotient Rule and many other calculus facts. However, there is an alternative to memorizing complicated formulas, such as (14), (16) on page 383, and very complicated formulas, such as (17), (18), (19) on page 383.

At the bottom of page 383, the authors include the following comment on the 19 formulas: **Although we include this table of trigonometric integrals, it makes more sense to understand how to obtain a given integral than to simply rely on the table.** Here is an example where we follow the authors' suggestion: Let us consider the problem $\int \sec^3 x dx$, which is a special case of $\int \tan^m x \sec^n x dx$ with m even and n odd. During an exam, you can use the integration technique of Section 7.1 of the textbook and write

$$\begin{aligned}\int \sec^3 x dx &= \int \sec^2 x \sec x dx = \tan x \sec x - \int \tan x (\tan x \sec x) dx \\ &= \tan x \sec x - \int (\sec^2 x - 1) \sec x dx \\ &= \tan x \sec x - \int \sec^3 x dx + \int \sec x dx.\end{aligned}$$

You would rewrite the above as

$$2 \int \sec^3 x dx = \tan x \sec x + \int \sec x dx.$$

The easy formula (13) on page 383 allows you to conclude

$$\begin{aligned}\int \sec^3 x dx &= (1/2) \tan x \sec x + (1/2) \int \sec x dx \\ &= (1/2) \tan x \sec x + (1/2) \ln |\tan x + \sec x| + C.\end{aligned}$$

Here is another example where we follow the authors' suggestion: In Example 10 (page 383) of Section 7.2, it is necessary to evaluate an integral of the type $\int \sin(mx) \cos(nx) dx$, where $m = 4$ and $n = 3$. Instead of using the memorized formula (18), you can simply use the precalculus (trigonometric) identity $\sin A \cos B = \frac{1}{2} \sin(A - B) + \frac{1}{2} \sin(A + B)$. You would write

$$\begin{aligned}\int \sin(mx) \cos(nx) dx &= \frac{1}{2} \int \sin((m - n)x) dx + \frac{1}{2} \int \sin((m + n)x) dx \\ &= -\frac{1}{2} \frac{\cos((m - n)x)}{m - n} - \frac{1}{2} \frac{\cos((m + n)x)}{m + n} + C.\end{aligned}$$