1. Suppose \( f(x) = \frac{3}{x+2} \). Use the definition of derivative to find \( f'(x) \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{3}{x + h + 2} - \frac{3}{x + 2} \]

\[
= \lim_{h \to 0} \frac{3(x + 2) - 3(x + h + 2)}{h(x + h + 2)(x + 2)} = \lim_{h \to 0} \frac{-3h}{h(x + h + 2)(x + 2)} \]

\[
= \lim_{h \to 0} \frac{-3}{(x + h + 2)(x + 2)} = \frac{-3}{(x + 2)^2}.
\]

2. Find an equation for the line tangent to the graph of \( y = \frac{4x}{2 + x^2} \) at the point where \( x = 1 \).

When \( x = 1 \), the value of \( y \) is \( 4/3 \).

\[
\frac{dy}{dx} = \frac{(2 + x^2)4 - (4x)(2x)}{(2 + x^2)^2} = \frac{8 - 4x^2}{(2 + x^2)^2}.
\]

When \( x = 1 \), this is \( 4/9 \). Thus an equation for the tangent is

\[
y - \frac{4}{3} = \frac{4}{9}(x - 1).
\]

3. Assume that the functions \( u(x) \) and \( v(x) \) are defined and differentiable for all real numbers \( x \). The following data is known about \( u, v, \) and their derivatives.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( u(x) )</th>
<th>( v(x) )</th>
<th>( u'(x) )</th>
<th>( v'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>-2</td>
</tr>
</tbody>
</table>

Define \( f(x) = u(x)v(x) \), \( g(x) = u(x)/v(x) \), and \( h(x) = u(v(x)) \). Give the values of the following with a brief indication of how they were obtained:

a) \( f'(2) \)

\[
f'(2) = u(2)v'(2) + u'(2)v(2) = 3 \cdot 2 + (-1) \cdot 4 = 2.
\]

b) \( g'(3) \)

\[
g'(3) = \frac{v(3)u'(3) - u(3)v'(3)}{v(3)^2} = \frac{1 \cdot 3 - 2 \cdot (-1)}{1^2} = 5.
\]
c) $h'(4)$

$$h'(4) = u'(v(4))v'(4) = u'(3)v'(4) = 3 \cdot (-2) = -6.$$  

(14) 4. Suppose that the function $f(x)$ is described by

$$f(x) = \begin{cases} 
3 - x^2 & \text{if } x < 0 \\
Ax + B & \text{if } 0 \leq x \leq 1 \\
2^x & \text{if } 1 < x 
\end{cases}.$$

a) Find $A$ and $B$ so that $f(x)$ is continuous for all numbers. Briefly explain your answer.

The value of $f(0)$ is $B$ and

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 3 - x^2 = 3.$$

If $f(x)$ is continuous at 0, then $B = 3$.

The value of $f(1)$ is $A + B = A + 3$ and

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2^x = 2.$$

Therefore $A + 3 = 2$ or $A = -1$.

b) Sketch $y = f(x)$ on the axes given for the values of $A$ and $B$ found in a) when $x$ is in the interval $[-2, 2]$. 

![Graph of the function](image-url)
(20) 5. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) \[ \lim_{x \to 1} \frac{x^2 + 2x - 3}{x - 1} \]

\[ \lim_{x \to 1} \frac{x^2 + 2x - 3}{x - 1} = \lim_{x \to 1} \frac{(x - 1)(x + 3)}{x - 1} = \lim_{x \to 1} x + 3 = 4. \]

b) \[ \lim_{x \to 2^+} \frac{|x - 1| - 1}{|x - 2|} \]

If \( x > 2 \), then both \( x - 1 \) and \( x - 2 \) are positive and \( |x - 1| = x - 1 \) and \( |x - 2| = x - 2 \). Therefore

\[ \lim_{x \to 2^+} \frac{|x - 1| - 1}{|x - 2|} = \lim_{x \to 2^+} \frac{x - 2}{x - 2} = \lim_{x \to 2^+} 1 = 1. \]

c) \[ \lim_{x \to 0} \frac{\sin 2x}{\tan 3x} \]

\[ \lim_{x \to 0} \frac{\sin 2x}{\tan 3x} = \lim_{x \to 0} \frac{\sin 2x}{\frac{\sin 3x}{\cos 3x}} = \lim_{x \to 0} \frac{\sin 2x \cos 3x}{\sin 3x} = \lim_{x \to 0} \frac{2 \sin 2x \cos 3x}{3 \sin 3x} = \frac{2}{3}. \]

d) \[ \lim_{x \to 4} \frac{3x - 2}{\cos(\pi x)} \]

\[ \lim_{x \to 4} \frac{3x - 2}{\cos(\pi x)} = \frac{3 \cdot 4 - 2}{\cos(4\pi)} = 10. \]

(10) 6. Suppose that \( f(x) \) is defined and continuous for all real numbers \( x \) and assume that \( f(x) \) takes on the following values: \( f(-2) = 6, f(0) = -3, f(2) = 4, f(3) = 0, f(4) = -1, f(7) = -3 \), and \( f(10) = 8 \).

a) What can be said about the number of solutions to the equation \( f(x) = 0 \)?

There are at least four solutions to the equation \( f(x) = 0 \).

b) Give a list of nonoverlapping intervals in which solutions to the equation \( f(x) = 0 \) can be found.

By the Intermediate Value Theorem there is at least one solution of the equation \( f(x) = 0 \) in each of the intervals \((-2, 0), (0, 2), [3, 3], \) and \((7, 10)\).

(8) 7. What is the domain of \( f(x) = \frac{\ln x + \sqrt{4-x}}{\sin x} \)? Give your answer as a list of intervals. Explain how you arrived at your answer.

\( \ln x \) is defined only for \( x > 0 \).

\( \sqrt{4-x} \) is defined only for \( x \leq 4 \).
\[ \frac{1}{\sin x} \] is defined only when \( x \) is not of the form \( n\pi \) for some integer \( n \).

The numerator of \( f(x) \) is defined for \( x \) in the interval \((0, 4]\). However, that interval contains one integer multiple of \( \pi \), namely \( \pi \) itself. Thus the domain of \( f \) consists of the two intervals \((0, \pi)\) and \((\pi, 4]\).

8. In this problem the function \( f(x) \) has domain the open interval \((-4, 4)\). A graph of \( y = f(x) \) is displayed below. Answer the following questions as well as you can based on the information in the graph.

\[ \begin{array}{|c|c|}
\hline
x & y \\
\hline
-4 & 4 \\
-3 & 3 \\
-2 & 2 \\
-1 & 1 \\
0 & 0 \\
1 & 0 \\
2 & 0 \\
3 & 1 \\
4 & 4 \\
\hline
\end{array} \]

(a) For which \( x \) is \( f(x) \) not continuous?

\[ x = -1 \]

(b) For which \( x \) is \( f(x) \) not differentiable?

\[ x = -1 \text{ and } x = 2 \]

(c) For which \( x \) is \( f'(x) = 0? \)

\[ x = -2 \]

(d) For which \( x \) is \( f'(x) > 0? \)

For those \( x \) with \( 0 < x < -2 \) or \( 2 < x < 4 \)
9. a) If \( f(x) = \frac{1 - e^x}{x^2 + 1} \), what is \( f'(x) \)?

\[
f'(x) = \frac{(x^2 + 1)(-e^x) - (1 - e^x)(2x)}{(x^2 + 1)^2}.
\]

b) If \( f(x) = (2x + 3\cos x)(x^4 - x^2) \), what is \( f'(x) \)?

\[
f'(x) = (2x + 3\cos x)(4x^3 - 2x) + (2 - 3\sin x)(x^4 - x^2).
\]

c) If \( f(x) = \sec(x^3 + 2x) \), what is \( f'(x) \)?

\[
f'(x) = \sec(x^3 + 2x)\tan(x^3 + 2x)(3x^2 + 2).
\]