Short solutions for some of the review problems for exam 1

1. Domain of $g(f(x))$ is $(-\infty, -8) \cup (5, \infty)$.

2. (a) -1/4, (b) 1/6, (c) 3/2.

3. Use Intermediate Value Theorem, after verifying that its hypotheses are satisfied.

4. The equation for the tangent line at $x_0 = 3$ is $y = y_0 + m(x - x_0)$ with $m = g'(3)$. So $g'(3) = -1/2$ and $y_0 = g(3) = 3/2$.

5. (a) C=3 (b) yes (c) 10, -1 (d) no (e) graph should have sharp angle at x=0, no angle at x=1 with a hole at (1,3) that is filled if C=3..

6. evaluate (f(x+h) - f(x))/h = 2x+ h, then take limit as h goes to zero, to get 2x.

7. many possibilities!

8. get dy/dx by implicit differentiation; then plug in x=2 and y=1 to get m = -5. Equation is then y-1 = (-5)(x-2), or y = -5x + 11.

9. 60

10. standard differentiation problems. Remember the chain rule!

(a) $f'(x) = (3/5)x^{-2/5} + 6(-3/8)x^{-11/8} + 3x^2 + 0$

(b) $g'(x) = (x + 3x^9)(5x^4 - 7) + (x^5 - 7x)(1 + 27x^8)$

(c) $h'(x) = 5\left(\frac{x^2+3x}{x^6-9x}\right)^4 \left(\frac{(x^6-9x)(2x+3)-(x^2+3x)(6x^5-9)}{(x^6-9x)^2}\right)$

(d) $k'(x) = (1/3)(x^7 + 5x^2 - 50)^{-2/3}(7x^6 + 10x)$

11. $(-5/4)x^{-3/2}$; 0

12. (a) $R(x) = 120x - .001x^2$

(b) $P(x) = -6,000 + 116x - .001x^2$

(c) $P'(x) = 116 - .002x$

(d) P'(4000) = 108 dollars. (Note that the question really should have asked for the profit on the 4,001 telescope, but the two numbers are close enough together.)

13. The quantity demanded is decreasing by 1/8 unit per week.

14. (a) -2, -3, DNE, 2, -2, DNE. DNE's both have different right and left limits.

(b) No, Yes, No, Yes, Yes, No. The extra No is because F(-3) is not defined.

(c) No, Yes (F'(-1) looks to be approximately 1/2), No, No, Yes (F'(6) = 0), No. The extra No is because of the angle in the graph at x=2.