Review Problems for the second midterm exam in Math 135 Fall 2001

NOTE : These are only practice problems!
The number of problems in the exam will be less than this review.
You are responsible to study **all** the material and should be able to do also **all** homework problems!
You can find additional problems in the 135 Fall 2000 webpage.

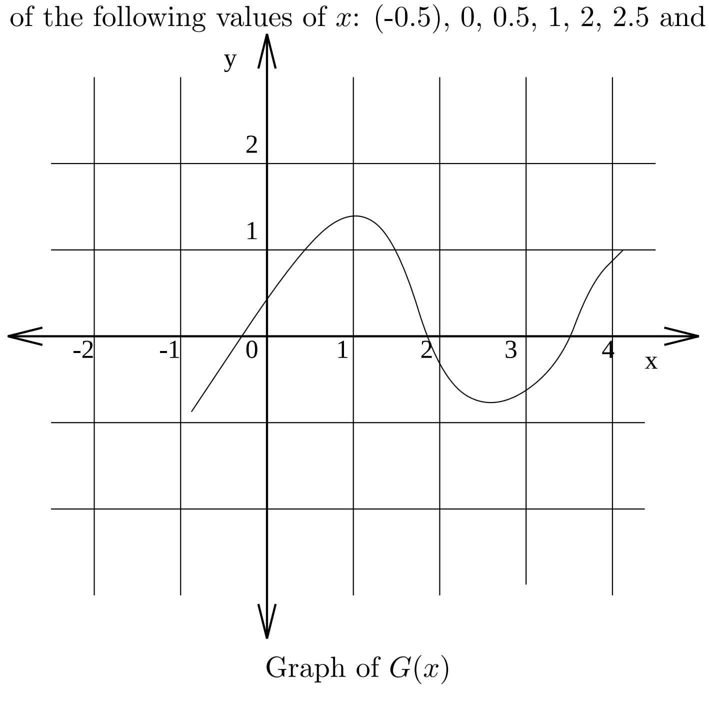
1.Sketch a graph of a continuous and a differentiable function $f(x)$ with the following properties: The only critical points of $f$ are $x = 0, \quad x = 2 \quad and \quad x = 6$ , **and** $f''(x) = 0$ only when $x = 1 \ \ and \ \ x = 4$ , **and** $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = -\infty$.

2. Let $f(x) = 2x^3 - 6x^2 - 18x + 3$.
a) Find all relative extrema of $f(x)$. Apply the second derivative test or the first derivative test to find which are relative minima and which are relative maxima.
b) Find the absolute maximum and absolute minimum of $f(x)$ on $[-2,\ 2]$.

3. An open box with a square base and a total volume of 100 cubic inches is to be constructed from two types of materials. The base should be made of a heavy duty metal which cost $4 per square inch and the sides should be made of a cardboard which cost 80 cents per square inch. Find the dimensions of the box that will minimize the total cost.
Don't forget to check that your answer **minimizes** the cost.

4. Sketch the graph of the function $f(x) = \dfrac{x^2-1}{x^2-4}$ using **calculus only!** (It is OK to use your calculator to check your answer, but you need to say, based on calculus, how the answers are obtained.)
**Show all work:** find the domain of $f(x)$, $f(0)$, and all vertical and horizontal asymptotes of $f(x)$ **with the appropriate limits**.
Find where the function is increasing/decreasing and where it is concave up and down.
Indicate on the graph all local extrema and all inflection points, if any.

5. Use the graph of the function $G(x)$ below, to find if $G(x), \ \ G'(x) \ \ and \ \ G''(x)$ are positive, negative or 0 at each of the following values of $x$: (-0.5), 0, 0.5, 1, 2, 2.5 and 3.




Graph of $G(x)$

6. Use logarithmic differentiation to compute the derivative of $\dfrac{(x^5+6x^2)^4(x^6+x^4+12)}{(x^2+5)^2x^8}$

7. Find the value of $x$ given that $2\ln x + 3\ln x = e^{\ln 32}$.

8. The **derivative** of $f$ is $f'(x) = x^{15}(x+1)^{10}$
a. Find all the critical points of $f$ and check where $f$ is increasing and decreasing.
b. Compute $f''(x)$ and find what are the $x$ values at the points of inflection.
Note : **Do not try to find f!**

9. Find the derivatives of the following functions :

$F(x) = \tan^2 x + e^x \sin 8x \qquad G(x) = \ln \dfrac{x^2+5}{x+6} + e^{x^3} \qquad H(x) = \cos x^2 + 5 + x^2e^x + \dfrac{1}{e}$

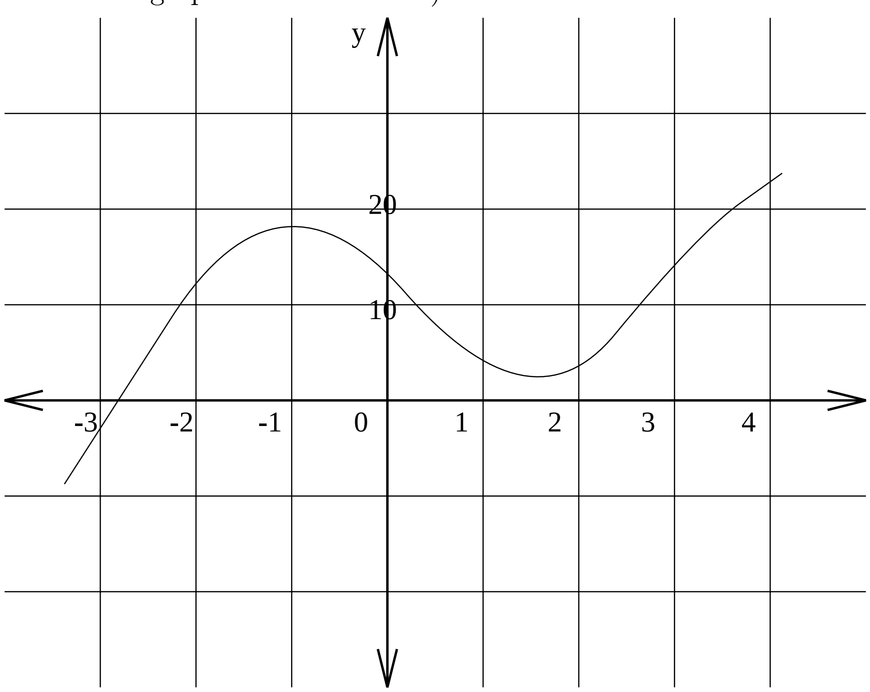
10. Find the absolute maximum and minimum of the function $g(x) = \sin x + \cos x$ on $[0\,,\,2\pi]$

11. Use differentials to approximate the values of $\sqrt[3]{8.01}$ and $\sqrt[3]{7.99}$.

12. The weekly quantity demanded of the "Comfy" chairs is given by $p + 0.2x^2 = 280$ where $p$ is measured in dollars and $x$ is number of chairs demanded per week. Use differentials to estimate the change in the price of a unit when the weekly quantity demanded changes from 20 to 21.

13. (a) Sketch the graph of $F(x) = x^2 + 1$ on $[0,\ 4]$.
(b) Show the Mean Value Theorem graphically (on your graph only) when $a = 0$ and $b = 4$.

14. Below is the graph of the derivative $f'(x)$ of a differentiable function $f$. Note that it is NOT the graph of $f$ itself!
a. Find all the critical points of $f$ (NOT $f'$), and for each check whether the function $f$ has a relative maximum or a relative minimum.
b. Find where the function $f$ is increasing and where it is decreasing.
c. Find where the function $f$ is concave up or down, and where are all the inflection points.
d. Use the information from a., b. and c. to sketch a graph of a possible function $f$ (so that the graph below is indeed the graph of its derivative).




Graph of $f'(x)$