Review Problems for the first exam in Math 135 Fall 2001

NOTE : These are only practice problems!

The number of problems in the exam will be less than this review.

You are responsible to study all the material and should be able to do also all homework problems!

- 1. Let $f(x) = \sqrt{x^2 + 3x 40}$, $g(x) = \frac{1}{x}$ a) Find g(f(x)) and f(g(x)). b) Find the domain of g(f(x)).
- 2. Find the value of each of the following limits in 3 different ways: analytically (exactly), graphically (illustrating the limit on a graph) and numerically (give a numerical evidence for the limit).

a)
$$\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 4}$$
 b) $\lim_{x \to 4} \frac{x^2 - 5x + 6}{x^2 - 4}$ c) $\lim_{x \to \infty} \frac{3x^4 + 10x + 7}{2x^4 - 5x^3}$

- 3. Let $g(x) = x^3 + x + 100$. Without graphing the function g, explain why there is at least one number $c \in (-5, 1)$ such that g(c) = 0. Don't try to find c!
- 4. The tangent line to the curve of some unknown function g(x) at x = 3 is 2y + x = 6. What is g(3)? What is g'(3)?.

5. Let

$$f(x) = \begin{cases} x^3 + 2 & x > 1\\ C & x = 1\\ 5x - 2 & 1 > x \ge 0\\ x^2 - 2 & x < 0 \end{cases}$$

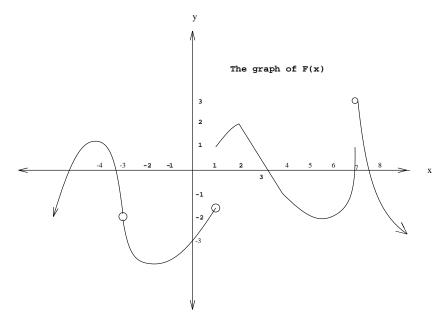
- a) For what value of C is f continuous at x = 1? Explain!
- b) Is f continuous at x = 0? Explain!
- c) Find the following limits : $\lim_{x\to 2} f(x)$, $\lim_{x\to (-1)} f(x)$
- d) Is f(x) differentiable at x = 0? Explain!.
- e) Check your answers to b) and d) by sketching the graph of f(x) on the interval [-1, 1).
- 6. Use the definition of the derivative as a limit to find the derivative of $f(x) = x^2 + 2$.
- 7. Sketch a possible graph of F on [-4, 4] such that: F is continuous on [-4, -1) and (-1, 4], $\lim_{x\to -1^-} F(x) = 5$, $\lim_{x\to -1^+} F(x) = 2$, F is not differentiable only at x = -1 and x = 1.
- 8. Find the equation of the tangent line to the curve given by the **implicit** function $y^2 xy = x^2 + y 6$ at (2, 1).
- 9. Let h(x) = f(g(x)). Assume that f'(2) = 5, g(1) = 2 and g'(1) = 12. What is h'(1)?
- 10. Find the derivative of the following functions. Don't simplify!

a)
$$f(x) = \sqrt[5]{x^3} + \frac{6}{x^{3/8}} + x^3 + 7$$
 b) $g(x) = (x + 3x^9) * (x^5 - 7x)$
c) $h(x) = \left(\frac{x^2 + 3x}{x^6 - 9x}\right)^5$ d) $k(x) = \sqrt[3]{x^7 + 5x^2 - 50}$

- 11. a) Find the second derivative of the function $F(x) = 5\sqrt{x}$ b) Find $\frac{d^9g}{dt^9}$ where $g(t) = 120t^7 + t^6 - 62t^4 + 930$
- 12. Suppose that the total monthly cost function (in dollars) associated with manufacturing x Starview brand telescopes is given by C(x) = 6,000 + 4x when $0 \le x \le 10,000$. The wholesale **unit** price is f(x) = 120 - 0.001x when x denotes the monthly quantity demanded.

Recall that the revenue is given by xf(x) and the profit by P(x) = R(x) - C(x).

- a) Find the daily revenue function R(x).
- b) Find the daily profit function P(x).
- c) Find the daily marginal profit function.
- d) Find the approximate actual profit realized from the sale of the 4,000th telescope.
- 13. The demand function for a Starview brand telescope cover is $p = 35 0.02x^2$ where p is the unit price in dollars and x is the quantity demanded each week. How fast is the quantity demanded changing when x = 100 and the unit price per cover is increasing at a rate of 50 cents per week?



14. Use the given graph of F to do a), b), and c) for x = a where a = -3, 0, 1, 2, 6, 7.

- a) What is $\lim_{x\to a} F(x)$? If the limit does not exist, write DNE and explain why.
- b) Is F continuous at x = a? If F is discontinuous at x = a, explain why.
- c) Is F differentiable at x = a? If F is not differentiable at x = a, explain why.