Formula Sheet For the Final Exam in Calculus 135 Fall 2001

Theorem: $\lim\limits_{x\to a} f(x) = L$ if and only if $\lim\limits_{x\to a^+} f(x) = \lim\limits_{x\to a^-} f(x) = L$

This theorem says that $\lim\limits_{x\to a} f(x)$ exists if and only if:

1. $\lim\limits_{x\to a^+} f(x)$ exists *and* 2. $\lim\limits_{x\to a^-}$ exists *and* 3. Both limits in 1. and 2. are equal.

Definition: f is **continuous** at a if the following conditions are satisfied :

1. $f(a)$ is defined. 2. $\lim\limits_{x\to a} f(x)$ exists 3. $\lim\limits_{x\to a} f(x) = f(a)$

Definition: $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ so $f'(a) = f'(x)\Big|_{x=a} = \lim\limits_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$

f is **differentiable** at a if $f'(a)$ exists.

Definition: The **differential dy** is: $dy = f'(x)dx$. Compare with $\Delta y = f(x+\Delta x) - f(x)$.
$f(a+\Delta x) \approx f(a) + dy = f(a) + f'(a)dx$

Graphing and Optimization. **Local max and min** of a function f can occur only at **critical points** (any point x in the domain of f such that $f'(x) = 0$ or $f'(x)$ does not exist). **Absolute max and min** occur only at critical points or endpoints. **Inflection points** are points $\big(x, f(x)\big)$ where the concavity of f changes and can occur only where $f''(x)$ is zero or does not exist. The line $x = a$ is a **vertical asymptote** of the graph of a function f if $\lim\limits_{x\to a^+} = +\infty$ or $= -\infty$, or if $\lim\limits_{x\to a^-} = +\infty$ or $= -\infty$,

The line $y = b$ is a **horizontal asymptote** of the graph of f if $\lim\limits_{x\to +\infty} f(x) = b$ or $\lim\limits_{x\to -\infty} f(x) = b$

Basic Fact: $f'(a)$ is the **slope of the tangent line** to the graph of f at $x = a$

Basic log and exp laws: $e^{\ln x} = x$ for $x > 0$ $\ln e^x = x$ for all real numbers x

$\ln(mn) = \ln m + \ln n$ $\ln \frac{m}{n} = \ln m - \ln n$ $\ln m^n = n \ln m$ $\ln 1 = 0$ $\ln e = 1$

$e^a e^b = e^{a+b}$ $\frac{e^a}{e^b} = e^{a-b}$ $(e^a)^b = e^{ba}$ $e^0 = 1$ $e^1 = e$

Basic Trig Identities: $\sin^2 x + \cos^2 x = 1$ $\sin(-x) = -\sin x$ $\cos(-x) = \cos x$

$\sin(2\pi + x) = \sin x$ $\cos(2\pi + x) = \cos x$ 360 degrees $= 2\pi$ radians.

$\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\csc x = \frac{1}{\sin x}$

Definition: $f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$ is a **Riemann sum** for a function f corresponding to a partition of the interval $[a, b]$ into n subintervals of equal width $\Delta x = \frac{(b-a)}{n}$, if x_1 is in the first subinterval, x_2 is in the second subinterval, etc.

Definition: The **definite integral of f from a to b**, written $\displaystyle\int_a^b f(x)\,dx$, is defined to be the limit as $n \to \infty$ of such Riemann sums, if the limit exists (for all choices of representative points $x_1, x_2, \ldots x_n$ in the n subintervals).

Thus, $\displaystyle\int_a^b f(x)dx = \lim_{n\to\infty} \big[\, f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x \,\big]$

Theorem: Let G be an antiderivative of f on an interval I. Then every antiderivative F of f on I must be of the form $F(x) = G(x) + C$ where C is a constant.

Fact: Let f be continuous and nonnegative on $[a, b]$, then $\displaystyle\int_a^b f(x)\,dx$ is equal to the **area of the region under the graph of f on** $[a, b]$. If f is sometimes negative on $[a, b]$ then $\displaystyle\int_a^b f(x)\,dx$ is equal to the **area of the region above** $[a, b]$ **minus the area of the region below** $[a, b]$.

The Fundamental Theorem of Calculus: Let f be continuous on the closed interval $[a,b]$, then $\displaystyle\int_a^b f(x)dx = F(b) - F(a)$ where F is any antiderivative of f (that is $F'(x) = f(x)$).

Definition: The **average value** of an integrable function f over $[a,b]$ is: $\displaystyle\frac{1}{b-a}\int_a^b f(x)dx$

Differentiation Rules	**and**	**Integration Rules**
$(kx)' = k$		$\int k\,dx = kx + C$
$\big(cf(x)\big)' = cf'(x)$		$\int cf(x)\,dx = c\int f(x)\,dx$
$(x^r)' = rx^{r-1}$		$\int x^r\,dx = \frac{x^{r+1}}{r+1} + C$ for $r \neq -1$
$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$		$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
$(e^x)' = e^x$		$\int e^x\,dx = e^x + C$
$(\ln x)' = \frac{1}{x}$		$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$(\sin x)' = \cos x$		$\int \cos x\,dx = \sin x + C$
$(\cos x)' = -\sin x$		$\int \sin x\,dx = -\cos x + C$
$(\tan x)' = \sec^2 x$		$\int \sec^2 x\,dx = \tan x + C$

Chain Rule: If $h(x) = g[f(x)]$, then $h'(x) = g'\big(f(x)\big) * f'(x)$ Equivalently, if we write $y = h(x) = g(u)$, where $u = f(x)$, then $\dfrac{dy}{dx} = \dfrac{dy}{du} * \dfrac{du}{dx}$

Integration by Substitution: Let $u = g(x)$ and $F(x)$ be the antiderivative of $f(x)$. Then $du = g'(x)dx$ and $\displaystyle\int f\big(g(x)\big)g'(x)\,dx = \int f(u)\,du = F(u) + C$

Also, $\displaystyle\int_a^b f\big(g(x)\big)g'(x)dx = \int_{g(a)}^{g(b)} f(u)\,du = F\big(g(b)\big) - F\big(g(a)\big)$

Product Rule: $[f(x) * g(x)]' = f'(x) * g(x) + g'(x) * f(x)$

Quotient Rule: $\displaystyle\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x) * f'(x) - f(x) * g'(x)}{g^2(x)}$

Properties of the Definite Integral

$\displaystyle\int_a^a f(x)\,dx = 0$ (same integration limits) $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ (exchange integration limits)

$\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ where $a < c < b$.

Calculus in Economics Definitions, from section 3.4:

The demand equation relates price per unit p and number of units x. It can be solved for p as a function of x, or x as a function of p. Revenue $R = px$. (Here usually price p is written as a function of x, using the demand equation, so that R becomes a function of x only.) Profit P equals revenue R minus total cost C. Average cost $\bar{C}(x) = \frac{C(x)}{x}$