135 first exam formula sheet. Any addition will void the use of this sheet!

Definition: $\lim_{x\to a} f(x) = L$ if the value f(x) can be made as close to the number L as we please by taking x sufficiently close to (but not equal to) a.

Theorem : $\lim_{x\to a} f(x) = L$ iff $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$. This theorem says that $\lim_{x\to a} f(x)$ exists iff :

1. $\lim_{x\to a^+} f(x)$ exists and 2. $\lim_{x\to a^-} f(x)$ exists and 3. Both limits in 1. and 2. are equal.

Definition: f is continuous at a if the following conditions are satisfied :

1. f(a) is defined.

- 2. $\lim_{x\to a} f(x)$ exists
- 3. $\lim_{x \to a} f(x) = f(a)$

The Intermediate Value Theorem :

If f is a continuous function on a closed interval [a, b] and M is any number between f(a) and f(b), then there is at least one number c in [a, b] such that f(c) = M.

Definitions:
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
 so $f'(a) = f'(x)|_{x=a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

f is **differentiable** at a if f'(a) exists.

Fact: f'(a) is the slope of the tangent line to the graph of f at x = a

Differentiation rules:

Product rule: [f(x) * g(x)]' = f'(x) * g(x) + g'(x) * f(x)

Quotient rule:
$$\left[\frac{f(x)}{g(x)}\right]' = \frac{g(x) * f'(x) - f(x) * g'(x)}{g^2(x)}$$

Power Rule: $(x^r)' = rx^{r-1}$

Chain Rule: If h(x) = g[f(x)], then h'(x) = g'(f(x)) * f'(x)

Equivalently, if we write y = h(x) = g(u), where u = f(x), then $\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$

Calculus in economics definitions, from section 3.4:

Here p is the price per unit and x the number of units. The demand equation relates p and x and can be solved for p as a function of x, or x as a function of p.

Revenue R = px. (Here usually p is written as a function of x, using the demand equation, so that R becomes a function of x only.)

Profit P equals revenue R minus total cost C.

Average cost $\bar{C}(x) = \frac{C(x)}{x}$

Elasticity of demand: if f is a differentiable demand function written as x = f(p), then the elasticity of demand at price p is given by $E(p) = -\frac{pf'(p)}{f(p)}$. The demand is **elastic** (price increase causes revenue to decrease) if E(p) > 1, **inelastic** (price increase causes revenue to increase) if E(p) < 1, and **unitary** if E(p) = 1.

Geometry: For a sphere of radius r, the volume is $\frac{4}{3}\pi r^3$ and surface area is $4\pi r^2$.