Definition: $\lim _{x \rightarrow a} f(x)=L$ if the value $f(x)$ can be made as close to the number $L$ as we please by taking $x$ sufficiently close to (but not equal to) $a$.
Theorem : $\lim _{x \rightarrow a} f(x)=L$ iff $\lim _{x \rightarrow a^{+}} f(x)=\lim _{x \rightarrow a^{-}} f(x)=L$.
This theorem says that $\lim _{x \rightarrow a} f(x)$ exists iff :

1. $\lim _{x \rightarrow a^{+}} f(x)$ exists and $2 . \lim _{x \rightarrow a^{-}} f(x)$ exists and 3 . Both limits in 1 . and 2 . are equal.

Definition: $f$ is continuous at $a$ if the following conditions are satisfied :

1. $f(a)$ is defined.
2. $\lim _{x \rightarrow a} f(x)$ exists
3. $\lim _{x \rightarrow a} f(x)=f(a)$

## The Intermediate Value Theorem :

If f is a continuous function on a closed interval $[a, b]$ and $M$ is any number between $f(a)$ and $f(b)$, then there is at least one number $c$ in $[a, b]$ such that $f(c)=M$.

Definitions: $f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} \quad$ so $\quad f^{\prime}(a)=\left.f^{\prime}(x)\right|_{x=a}=\lim _{h \rightarrow 0} \frac{f(a+h)-f(a)}{h}$
$f$ is differentiable at $a$ if $f^{\prime}(a)$ exists.
Fact: $f^{\prime}(a)$ is the slope of the tangent line to the graph of $f$ at $x=a$

## Differentiation rules:

Product rule: $[f(x) * g(x)]^{\prime}=f^{\prime}(x) * g(x)+g^{\prime}(x) * f(x)$
Quotient rule: $\left[\frac{f(x)}{g(x)}\right]^{\prime}=\frac{g(x) * f^{\prime}(x)-f(x) * g^{\prime}(x)}{g^{2}(x)}$
Power Rule: $\left(x^{r}\right)^{\prime}=r x^{r-1}$
Chain Rule: If $h(x)=g[f(x)]$, then $h^{\prime}(x)=g^{\prime}(f(x)) * f^{\prime}(x)$
Equivalently, if we write $y=h(x)=g(u)$, where $u=f(x)$, then $\frac{d y}{d x}=\frac{d y}{d u} * \frac{d u}{d x}$
Calculus in economics definitions, from section 3.4:
Here $p$ is the price per unit and $x$ the number of units. The demand equation relates $p$ and $x$ and can be solved for $p$ as a function of $x$, or $x$ as a function of $p$.
Revenue $R=p x$. (Here usually $p$ is written as a function of $x$, using the demand equation, so that $R$ becomes a function of $x$ only.)
Profit $P$ equals revenue $R$ minus total cost $C$.
Average cost $\bar{C}(x)=\frac{C(x)}{x}$
Elasticity of demand: if $f$ is a differentiable demand function written as $x=f(p)$, then the elasticity of demand at price $p$ is given by $E(p)=-\frac{p f^{\prime}(p)}{f(p)}$. The demand is elastic (price increase causes revenue to decrease) if $E(p)>1$, inelastic (price increase causes revenue to increase) if $E(p)<1$, and unitary if $E(p)=1$.
Geometry: For a sphere of radius $r$, the volume is $\frac{4}{3} \pi r^{3}$ and surface area is $4 \pi r^{2}$.

