# DEFINITE INTEGRALS

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The question of evaluating definite integrals is as old as calculus itself. In spite of that, there is no coherent theory that will tell us how to proceed when confronted with a specific integrand.

In this talk I will discuss two aspects of this problem. The first one deals with a series of transformations on the parameteres of a rational integrand. This is a rational version of the classical transformation of Landen, Gauss and Legendre for an elliptic integral that produced the arithmetic-geometric mean. The simplest example relates

$$\int_0^\infty \frac{dx}{ax^2+bx+c}$$

to the dynamics of

$$\begin{aligned}
a_{n+1} &= a_n \left[ \frac{(a_n+3c_n)^2 - 3b_n^2}{(3a_n+c_n)(a_n+3c_n) - b_n^2} \right], \\
b_{n+1} &= b_n \left[ \frac{3(a_n-c_n)^2 - b_n^2}{(3a_n+c_n)(a_n+3c_n) - b_n^2} \right], \\
c_{n+1} &= c_n \left[ \frac{(3a_n+c_n)^2 - 3b_n^2}{(3a_n+c_n)(a_n+3c_n) - b_n^2} \right],
\end{aligned}$$

with $a_0 = a$, $b_0 = b$ and $c_0 = c$.

The second problem illustrates the initial stages of a *homogeneity conjecture*. We present a case study on the family

$$L_n = \int_0^1 \ln^n \Gamma(q)\, dq$$

Euler evaluated

$$L_1 = \ln\sqrt{2\pi}$$

and in joint work with O. Espinosa we obtained

$$\int_0^1 (\ln\Gamma(q))^2\, dq = \frac{\gamma^2}{12} + \frac{\pi^2}{48} + \frac{1}{3}\gamma \ln\sqrt{2\pi} + \frac{4}{3}\left(\ln\sqrt{2\pi}\right)^2 - (\gamma + 2\ln\sqrt{2\pi})\frac{\zeta'(2)}{\pi^2} + \frac{\zeta''(2)}{2\pi^2}.$$

The evaluation of $L_3$ is still open.

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