Defining sets from graph coloring to Latin squares and Sudoku

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In a given graph G a set of vertices S with an assignment of colors is called a defining set (of a k-coloring), if there exists a unique extension of the colors of S to a proper k-coloring of the vertices of G. A defining set with minimum cardinality is called a minimum defining set and its cardinality, the defining number, is denoted by $d(G,k)$.

Defining sets are defined and discussed for many concepts and parameters in graph theory and combinatorics. For example in Latin squares a critical set is a partial Latin square that has a unique completion to a Latin square, and is minimal with respect to this property. Smallest possible size of a critical set in any Latin square of order n is of interest and is denoted by $scs(n)$. But any $n \times n$ Latin square may be used as an n-coloring of the Cartesian product of $K_n \square K_n$ and vice versa. The following conjecture which is made in 1995, is still open:

For any n, $d(K_n \square K_n, n)(= scs(n)) = \left\lfloor \frac{n^2}{4} \right\rfloor$.

Another example is Sudoku, a combinatorial number-placement puzzle, which has become a favorite pastimes of many all around the world. This is very similar to the notion of critical sets for Latin squares, or more generally, defining sets for graph colorings. We discuss these concepts in different areas and introduce some more open problems. Our emphasis will be in graph colorings and perfect matching. For some references and a survey, we refer to the author's webpage and papers referenced in there.