Due at the beginning of class, Monday, September 15, 2008

Please write careful, clear, and correct proofs of the statements using complete English sentences. Try to avoid both excessive length and excessive brevity.

1. (5 points) Suppose we define Dedekind cuts for the real numbers, $\mathbb{R}$, imitating *exactly* the definitions for $\mathbb{Q}$. Explain what the resulting object "is"? Is it a field? If it is, which field "is" it?

2. (5 points) Prove that the irrational numbers are dense in $\mathbb{R}$. That is, if x and y are any real numbers with $x < y$, then there is an irrational number w with $x < w < y$.

Hint You may use the fact referenced in class that no rational number has square equal to 2, and the theorem proved which shows that there is a positive real number with square equal to 2.

Textbook problems From chapter 1: problems 5, 7, 9, and 13. All are worth 5 points except for #7, which is worth 10.