Math 411 **9/1/2008**

Entrance "exam"

Due at the beginning of class, Monday, September 8, 2008

Please write careful, clear, and correct proofs of the statements using complete English sentences. Try to avoid both excessive length and excessive brevity.

1. (8 points) Suppose n is a positive integer. Prove that the product of any n consecutive positive integers is divisible by $n!$.

2. Suppose A is a non-empty subset of the positive integers, L is a real number, and $\{a_n\}$ is a sequence (a sequence is a real-valued function whose domain is the positive integers, $\mathbb{N}$). Then $\lim_{n \in A} a_n = L$ means: for all $\varepsilon > 0$ there is N in A so that if n is in A and $n > N$, then $|a_n - L| < \varepsilon$.

a) (2 points) If A is a finite non-empty set, then $\lim_{n \in A} a_n = L$ for all sequences $\{a_n\}$ and for all real numbers L.

b) (6 points) Suppose $A_1, A_2, \ldots, A_k$ is a pairwise disjoint decomposition of the positive integers into infinite subsets A_j with $1 \leq j \leq k$. That is, each of the A_j's is an infinite subset of the positive integers and each positive integer is in exactly one of the A_j's. Prove that $\lim_{n \in \mathbb{N}} a_n = L$ if and only if $\lim_{n \in A_j} a_n = L$ for all j.

c) (6 points) Is a statement similar to b) true if the positive integers are written as a union of an infinite number of pairwise disjoint infinite subsets? Either prove such a statement or give a counterexample.

Rules Please treat this as any other homework assignment. That is, you may consult textbooks or acquaintances or me (!), but the written work you hand in must be your own.